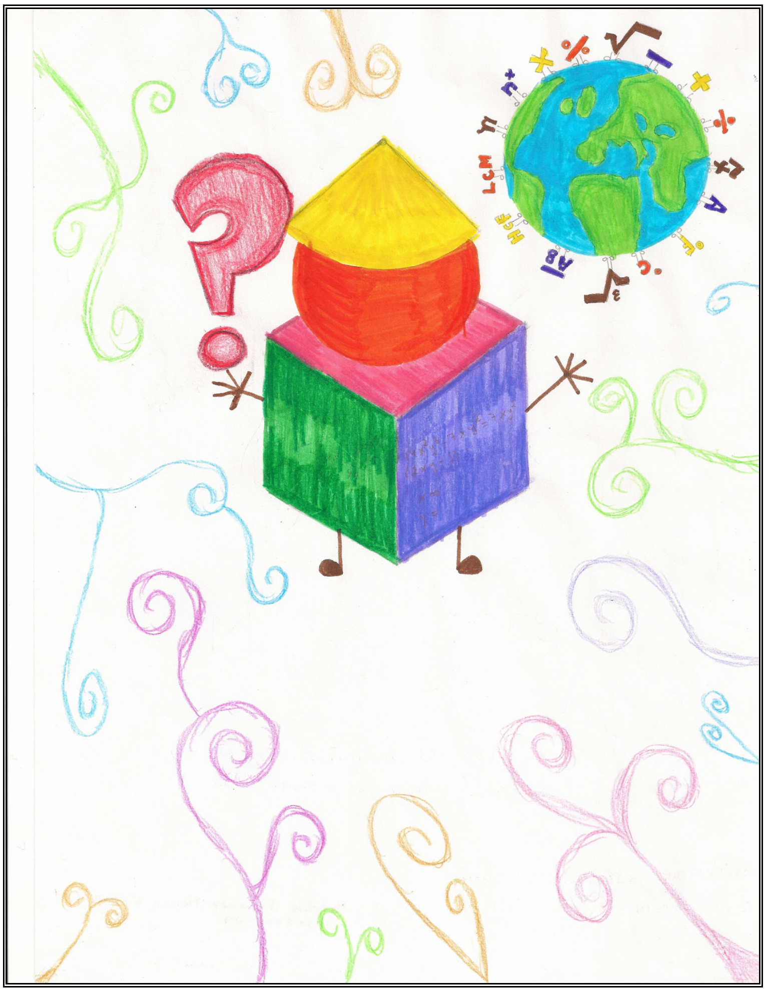
MATHEMATICS

1º ESO



PROGRAMA DE ENSEÑANZA BILINGÜE
I.E.S. ANDRÉS DE VANDELVIRA

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1º ESO B curso 2008-2009

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1 Numbers

1 Numbers

The **cardinal numbers** (one, two, three, etc.) are adjectives referring to **quantity**, and the **ordinal numbers** (first, second, third, etc.) refer to **distribution**.

Number	Cardinal	Ordinal
1	one	first (1^{st})
2	two	second (2^{nd})
3	three	third (3^{rd})
4	four	fourth (4^{th})
5	five	fifth
6	six	sixth
7	seven	seventh
8	eight	eighth
9	nine	ninth
10	ten	tenth
11	eleven	eleventh
12	twelve	twelfth
13	thirteen	thirteenth
14	fourteen	fourteenth
15	fifteen	fifteenth
16	sixteen	sixteenth
17	seventeen	seventeenth
18	eighteen	eighteenth
19	nineteen	nineteenth
20	twenty	twentieth
21	twenty-one	twenty-first
22	twenty-two	twenty-second
23	twenty-three	twenty-third
24	twenty-four	twenty-fourth
25	twenty-five	twenty-fifth
26	twenty-six	twenty-sixth
27	twenty-seven	twenty-seventh
28	twenty-eight	twenty-eighth
29	twenty-nine	twenty-ninth

30	thirty	thirtieth
40	forty	fortieth
50	fifty	fiftieth
60	sixty	sixtieth
70	seventy	seventieth
80	eighty	eightieth
90	ninety	ninetieth
100	one hundred	hundredth
1,000	one thousand	thousandth
100,000	one hundred thousand	hundred thousandth
1,000,000	one million	millionth

Beyond a million, the names of the numbers differ depending where you live. The places are grouped by thousands in America and France, by the millions in Great Britain, Germany and Spain.

Name	American-French	English-German-Spanish
million	1,000,000	1,000,000
billion	1,000,000,000 (a thousand millions)	1,000,000,000,000 (a million millions)
trillion	1 with 12 zeros	1 with 18 zeros
quadrillion	1 with 15 zeros	1 with 24 zeros

2 More about reading numbers

AND is used before the last two figures (tens and units) of a number.

325: three hundred **and** twenty-five
4,002: four thousand **and** two

A and ONE

The words *hundred*, *thousand* and *million* can be used in the singular with "*a*" or "*one*", but not alone. "*A*" is more common in an informal style; "*one*" is used when we are speaking more precisely.

I want to live for **a** hundred years
The journey took exactly **one** hundred years
I have **a** thousand euros

"A" is also common in an informal style with *measurement-words*

A kilo of oranges costs **a** euro
Mix **one** litre of milk with **one** kilo of flour…

"A" is only used with *hundred, thousand,* etc at the beginning of a number

146 **a** hundred and forty-six
3,146 three thousand, **one** hundred and forty-six

We can say "*a thousand"* for the round number 1,000, and we can say *"a thousand"* before *"and"*, but we say "*one thousand"* before a number of hundreds.

1,000 **a** thousand
1,031 **a** thousand and thirty-one
1,100 **one** thousand, one hundred
1,498 **one** thousand, four hundred and ninety-eight

Compare also:

A metre *but* one metre seventy (centimetres)
A euro *but* one euro twenty (cents)

Exercises I

1. Write in words the following numbers:

37 →________________________	27 →________________________
28 →________________________	84 →________________________
62 →________________________	13 →________________________
15 →________________________	158 →________________________
38 →________________________	346 →________________________
89 →________________________	461 →________________________
35 →________________________	703 →________________________
73 →________________________	102 →________________________
426 →________________________	1,870 →________________________
363 →________________________	1,015 →________________________
510 →________________________	1,013 →________________________
769 →________________________	6,840 →________________________
468 →________________________	8,900 →________________________
686 →________________________	6,205 →________________________
490 →________________________	9,866 →________________________
671 →________________________	7,002 →________________________

804 →________________________ 5,676 →________________________

3,750 →________________________ 77→________________________

3 [0] nought, zero, o, nil, love

The figure 0 is normally called **nought** in UK and **zero** in USA

- When numbers are said figure by figure, 0 is often called like the **letter O**

Examples:
My telephone number is nine six seven double two **o** four six **o** (967 220460)
My telephone number is nine six seven double two **o** treble/triple six (967 220666)

- In measurements (for instance, of temperature), 0 is called **zero**
Water freezes at **zero** degrees Celsius

- Zero scores in team-games are usually called **nil** in UK and **zero** in USA.

- In tennis, table-tennis and similar games the word **love** is used (this is derived from the French l'oeuf, meaning the egg, presumably because zero can be egg-shaped)

Examples:
Albacete three Real Madrid **nil (nothing)**
Nadal is winning forty-**love**

2. Write in words and read the following telephone numbers:

967252438	
678345600	
961000768	
918622355	
0034678223355	
0034963997644	

4 Decimals

Decimal fractions are said with each figure separate. We use a full stop (called "**point**"), not a comma, before the fraction. Each place value has a value that is one tenth the value to the immediate left of it.

0.75 (nought) **point** seventy-five or seventy-five hundredths

3.375 three **point** three seven five

5 Fractions and percentages

Simple fractions are expressed by using *"ordinal numbers"* (third, fourth, fifth...) with some exceptions:

1/2	One half / a half
1/3	One third / a third
2/3	Two thirds
3/4	Three quarters
5/8	Five eighths
4/33	Four **over** thirty-three

Percentages:

We don't use the article in front of the numeral

10% of the people	Ten **per cent** of the people

6 Roman numerals

I=1	(I with a bar is not used)
V=5	$\overline{V}$=5,000
X=10	$\overline{X}$=10,000
L=50	$\overline{L}$=50,000
C=100	$\overline{C}$ = 100 000
D=500	$\overline{D}$=500,000
M=1,000	$\overline{M}$=1,000,000

Examples:

1 = I	11 = XI	25 = XXV
2 = II	12 = XII	30 = XXX
3 = III	13 = XIII	40 = XL
4 = IV	14 = XIV	49 = XLIX
5 = V	15 = XV	50 = L
6 = VI	16 = XVI	51 = LI
7 = VII	17 = XVII	60 = LX
8 = VIII	18 = XVIII	70 = LXX
9 = IX	19 = XIX	80 = LXXX
10 = X	20 = XX	90 = XC
	21 = XXI	99 = XCIX

- There is no zero in the Roman numeral system.

- The numbers are built starting from the largest number on the left, and adding smaller numbers to the right. All the numerals are then added together.

- The exception is the subtracted numerals, if a numeral is before a larger numeral; you subtract the first numeral from the second. That is, IX is 10 - 1= 9.

- This only works for one small numeral before one larger numeral - for example, IIX is not 8; it is not a recognized roman numeral.

- There is no place value in this system - the number III is 3, not 111.

7 Decimal notation and place value

Every digit represents a different value depending on its position. For example in 54 the digit 5 represents fifty units, in 5329 the digit 5 represents five thousand units.

3. Write in words the following numbers as in the examples:

BILLION	HUNDRED MILLION	TEN MILLION	**MILLION**	HUNDRED THOUSAND	TEN THOUSAND	**THOUSAND**	HUNDRED	TEN	UNIT
8	3	4	1	6	7	2	9	3	4
				5	8	3	4	0	0

- “Eight billion three hundred forty one million six hundred seventy two thousand nine hundred and thirty four”.
- “Five hundred eighty three thousand four hundred”.

2,538 ____

90,304 ____

762 ____

8,300,690,285 ____

____

593 ____

1,237,569 ____

3,442,567,321 ____

____

76,421 ____

90,304 ____

762 ____

8,321,678____

____

250,005 ____

4. Read the following numbers:

120,000.321	453,897	700,560	5,542,678,987
34,765	94,540	345,971	82,754
763,123	1,867,349	500,340	4,580,200,170

5. Read the following numbers:

8,300,345   3,000,000,000   678,987,112   30,000,000,000

678,234,900

Use this table only if you need it.

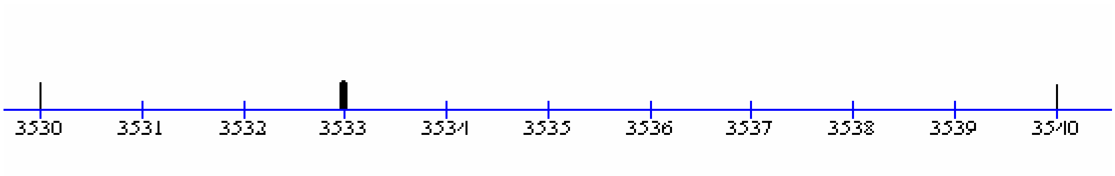
BILLION	HUNDRED MILLION	TEN MILLION	**MILLION**	HUNDRED THOUSAND	TEN THOUSAND	**THOUSAND**	HUNDRED	TEN	UNIT

8 Rounding numbers

When we use big numbers it is sometimes useful to approximate them to the nearest whole number

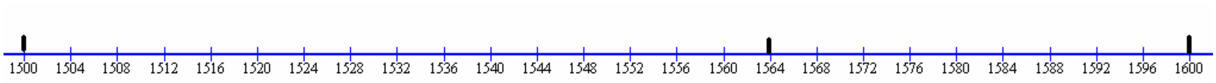
Examples:

1. Round 3533 to the nearest ten



3533 is closer to 3530 than 3540 so 3533 rounded to the nearest ten is 3530

2. Round 1564 to the nearest hundred



1564 is closer to 1600 than 1500 so 1564 rounded to the nearest hundred is 1600

The rule is:

1. Look at the digit which is one place on the right to the required approximation.
2. If the digit is less than 5, **cut the number** (change the digits on the right to zeros) as in the example 1.
3. If the digit is 5 or more than 5, **add one unit to the digit of the rounding position** and change the others to zeros like in the example 2.

Exercises II

1 Use the information of the table below to round the <u>population</u> to the nearest

- **a) Ten**
- **b) Hundred**
- **c) Ten thousand**

Round <u>the land areas</u> to the nearest

- **a) Hundred**
- b) **Thousand**

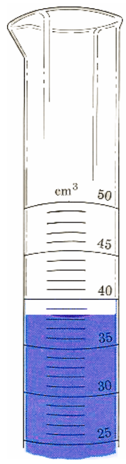
City/Land	Population	a)	b)	c)	Area (km^2)	a)	b)
Oxford	151,573				2605		
Worcester	93,353				1,741		
Edinburgh	451,710				263		
Hereford	50,468				2,180		
Glasgow	611,440				175		
Bristol	410,950				2,187		
London	7,355,420				1,577		
York	193,268				272		

2 Round the following numbers to the nearest indicated in the table

Numbers	Ten	Hundred	Thousand
6,172			
18,776			
5,217			

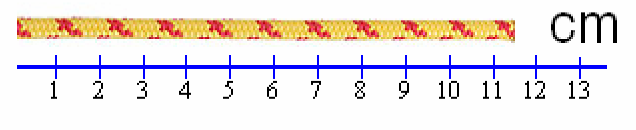
126,250			
5,208			
37,509			
8,399			
7,257			
129,790			
999			

3 Write the answer in the following cases:



a) What is the volume of liquid in the graduated cylinder to the nearest 10 ml?

b) How long is the rope to the nearest cm?



c) What is the weight of the bananas rounded to the nearest 100g and to the nearest kg?

d) If the capacity of this stadium is 75,638 people, round it appropriately to the nearest.

Ten

Hundred

Thousand

Rounding helps us to estimate the answers to calculations

4 For each question

a) **Estimate the answer by rounding each number appropriately.**
b) **Find the exact answer.**
c) **Check that both answers are similar.**

4.1 Anne bought a house for 76,595€ in 2001 and in 2007 sold it for 92,428€. Which was the profit?

a)

b)

c)

4.2 In a shoe shop 3,670 boxes of shoes have to be organized. There are three employees at the shop. How many boxes does each employee have to organize?

a)

b)

c)

4.3 Constance bought some furniture. She bought an armchair for € 499, a bed for 298€, a table for 189€ and four chairs at 97€ each. If she had a discount of 48€, how much did she have to pay?

a)

b)

c)

4.4 The “Instituto Andrés de Vandelvira” has 1,048 students, who have been distributed in groups of 30. How many are there in each group?

a)

b)

c)

4.5 The number of votes for each party in an election was: Party A 20,446, party B 10,866, party C 7,994 and others 5,743.

How many people voted?

What was the difference between the highest and the lowest numbers of votes?

a)

b)

c)

9 Calculations

What’s …? ¿cuánto es..? / ¿cuántos son..? **It’s** ... es… /son…

Addition

PLUS

In small additions we say ***and*** for **+** and ***is/are*** for **=**

2+6=8 two **and** six **are** eight

What’s eight **and** six? **It’s** eight

In larger additions (and in **more formal style**) we use ***plus*** for **+**, and ***equals*** or ***is*** for **=**

720 + 145= 865 Seven hundred **plus** two hundred **equals** / **is** nine

Subtraction **MINUS**

With small numbers, people say

7 - 4 = 3 four **from** seven **leaves/is** three *or* seven **take away** four **leaves/is** three

In a more formal style, or with larger numbers, we use ***minus*** and ***equals***

510 - 302 = 208 Five hundred and ten **minus** three hundred and two **equals** /**is** two hundred and eight

Multiplication **TIMES** **MULTIPLIED BY**

In small calculations we say

3 x 4 = 12 three fours **are** twelve

6 x 7 = 42 six sevens **are** forty-two

In larger calculations we can say

17 x 381 = 6,477 17 **times** 381 **is**/**makes** 6,477, or in a more formal style 17 **multiplied by** 381 **equals** 6,477

Division **DIVIDED BY**

270:3 = 90 Two hundred and seventy **divided by** three **equals** ninety

But in smaller calculations (8:2 = 4) we can say two **into** eight **goes** four (times)

Exercises III

1 Write the missing words. Write the answers in words

Twelve minus seven equals ____________

Six times five equals ____________

Eighty minus seventeen is _______________

Forty four minus nine plus twenty three equals_________________

Three times fifteen divided by five equals______________________

2 Write the missing numbers and write the answers in words as in the example

3 +14 = 17 three plus fourteen equals seventeen

1. **6x ___ = 42**
2. **18 ___ = 11**
3. **6: ____ + 7 = 10**
4. **12x3 - ____ = 25**
5. **(5x___ +5) : 8 = 5**

3 Write the missing operation symbols. Then write the answers in words

1. 6 __ 7__ 2 = 40

2. 8 __ 2 __ 5 =2

3. 28 __ 9 __ 1 = 18

4. 9 __ 3 __ 5 = 8

5. 49 __ 7 __ 3 = 10

6. 6 __ 4 __ 2 __ 8 = 0

Exercises IV

1 A shop is open daily except on Sundays. The profit after a year was 96300€.

a) Calculate the average (mean) per working day. (Total profit divided by the number of days)
b) Tony has worked in the shop every day for a year earning 294€ per week.

How much has he earned in a year?

How much per day?

2 A car travels 17 km per litre of petrol. How many litres are needed to travel 560 km? If the capacity of the tank is 42 litres how far can the car travel on a full tank?

3 Find three consecutive numbers whose product is 4080.

4 Calculate:

a) $48 \div (3+5)$

b) $(5+4) \times 14$

c) $(40+30) \div 5$

d) $(27+21) \div 3$

e) $(22+33) \div 11$

f) $(40 \div 20) \cdot 3$

5 Calculate:

a) $3+6 \cdot 2+5$

b) $(4+3) \cdot 5-2$

c) $15-6:2 \times 4$

d) $15-16:(3+1)$

e) $3+6 \times 2+10$

f) $(58-18) \cdot (27+13)$

g) $(32-8):(6-3)$

h) $(32-8):6-3$

i) $67+16 \times 3$

6 Insert brackets to make the following calculations correct

a) $5+4x8=37$

b) $5+4x8=72$

c) $6+15 \div 3=11$

d) $6+15:3=7$

e) $5+4+3x7=54$

f) $16+3x2+5=37$

g) $24/4+2 \cdot 7=28$

h) $240:5+7-4x3=8$

7 Abel buys 35 litres of petrol at 0.98€ per litre.
a) Estimate how much that costs by rounding appropriately.
b) Find the exact answer.
c) Check that both answers are similar.

2 Powers

1 Power

It is a number obtained by multiplying a number by itself a certain number of times

For example $3^4 = 3x3x3x3 = 81$

How to name powers.

6^5 Is read as

- **Six to the fifth power**
- **Six to the power of five**
- **Six powered to five**.

The most common is **six to the power of five**

6 is the **base**

5 is the **index** or **exponent**

Especial cases: **Squares and cubes** (powers of two and three)

3^2 is read as:

- **Three to the second power**
- **Three squared**
- **Three to the power of two.**
- **Three to the square power**

The most common is **Three squared**

5^3 Is read as:

- **Five cubed**
- **Five to the third power**
- **Five to the power of three**.

The most common is **Five cubed**

Exercises

1. Calculate mentally and write in words the following powers:

a) $4^3 =$ ____________________

b) $5^4 =$ ____________________

c) $11^2 =$ ____________________

d) $2^5 =$ ____________________

e) $5^3 =$ ____________________

f) $100^2 =$ ____________________

g) $10^3 =$ ____________________

2. Match the following numbers to their squares.

169 196 25 81 400 10000

20^2 13^2 9^2 5^2 100^2 14^2

3. Describe the pattern formed by the last digit of any square number.

- **Are there any numbers that do not appear as the last digits?**

- **Could** 413 **be a square number?**

4. Match the following numbers to their cubes.

125 8000 1 64 1000 27

20^3 4^3 10^3 3^3 5^3 1^3

5. Find and read the whole expression:

a) $3^3 =$ **b)** $6^3 =$ **c)** $600^2 =$ **d)** $2^7 =$ **e)** $1000^3 =$

6. Fill in the missing numbers.

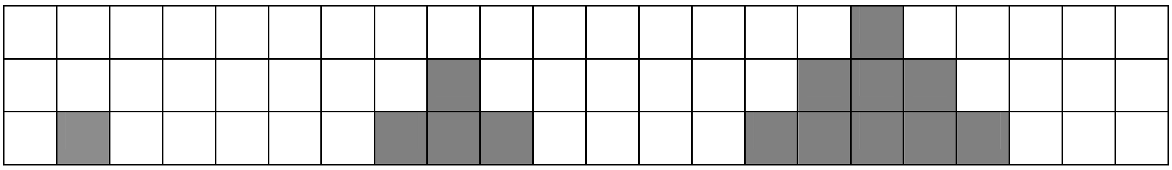
a) $5x5x5x5xx5 = 5^{[\ ]}$ **b)** $8x8x8x8 = 8^{[\ ]}$ **c)** $10000000 = 10^{[\ ]}$

d) $81 = [\ ]^4$ **e)** $16 = [\ ]^2$ **f)** $16 = 2^{[\ ]}$

7. Write a list with the squares of all the whole numbers from 1 to 12

8. Write a list with the cubes of all the whole numbers from 1 to 10

9. You can build up a pattern using square tiles.



Shape 1 **Shape 2** **Shape 3**

a) Draw the next two shapes in the pattern.

b) Count the numbers of tiles in each shape and put your results in a table

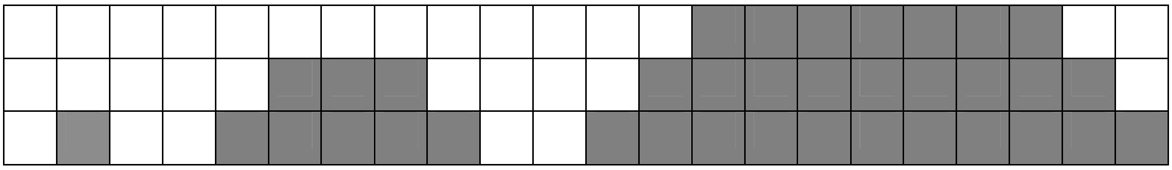
Shape number	**1**	**2**	**3**	**4**	**5**	
Tiles						

c) How many tiles would be in:

1. Shape 6 **2. Shape 9** **3. Shape 15**

d) Without drawing it, explain how to know the number of tiles when you know the number of the shape.

10. This pattern is built up using square tiles.



1 3 + 5 7+9+11

Shape 1 **Shape 2** **Shape 3**

e) Draw the next two shapes in the pattern.

f) Count the numbers of tiles in each shape and put your results in a table

Shape number	**1**	**2**	**3**	**4**	**5**	
Tiles						

g) How many tiles would be in:

1. Shape 5 **2. Shape 7** **3. Shape 10**

h) Without drawing it, explain how to know the number of tiles when you know the number of the shape.

11. Some numbers are equal to the sum of two squares, for example

$$3^2 + 5^2 = 34.$$

Which numbers smaller than 100 are equal to the sum of two squares?

How many of them are equal to the sum of two squares in more than one way?

Big numbers can be written using powers of 10, for example $70000 = 7x10^4$**,** $123000000 = 1.23x10^8$ this form of writing numbers is called **standard form.**

Exercise 12. Express in standard form the following numbers:

a) **4,000,000,000**

b) **A billion**

c) **321,650,000 (round to the million)**

d) **The length of the earth meridian in metres (round appropriately)**

e) **The number of seconds in a year (round appropriately)**

Exercise 13. Write as ordinary numbers

a) $3.4x10^5$ **b)** $0.05x10^2$ **c)** $2.473x10^8$

d) $7.26x10^2$ **e)** $7.006x10^7$ **f)** $9x10^{12}$

2 Operations with powers

To manipulate expressions with powers we use some rules that are called laws of powers or **laws of indices**.

2.1 Multiplication:

When powers **with the same base** are multiplied, the base remains unchanged and the **exponents are added**

Example:

$$7^5 x7^3 = (7x7x7x7x7)x(7x7x7) = 7^8$$

So $7^5 x7^3 = 7^8$

Exercise 14. Fill in the missing numbers.

a) $3^3 \cdot 3^4 = (3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3) = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^{[\ ]}$

b) $7^5 \cdot 7^8 = 7^{[\ \ ]}$ **c)** $6^2 \cdot 6^7 = 6^{[\ ]}$ **d)** $6^5 \cdot 6^{[\ \ ]} = 6^9$

e) $[\ \ ]^3 \cdot [\ \ ]^4 = 2^7$ **f)** $2^5 \cdot 2^{[\ \ ]} = 2^6$ **g)** $2^7 \cdot [\ \ ]^{[\ \ ]} = 2^9$

2.2 Division:

When powers **with the same base** are divided, the base remains unchanged and the **exponents are subtracted.**

Example:

$$6^7 : 6^4 = \frac{6x6x6x\cancel{6}x\cancel{6}x\cancel{6}x\cancel{6}}{\cancel{6}x\cancel{6}x\cancel{6}x\cancel{6}} = 6x6x6 = 6^3$$

So $6^7 : 6^4 = 6^3$

Exercise 15. Fill in the missing numbers.

a) $7^5 : 7^2 = 7^{[\ \ ]}$ **b)** $12^{13} : 127^7 = 12^{[\ \ ]}$ **c)** $3^8 : [\ \ ]^3 = 3^5$

d) $5^{[\ \ ]} : [\ \ ]^2 = 5^7$ **e)** $3^8 : [\ \ ]^3 = 3^5$ **f)** $[\ \ ]^{12} : [\ \ ]^9 = 9^{[\ \ ]}$

2.3 Power of a power:

The exponents or indices must be multiplied

Example:

$$\left(2^3\right)^5 = \left(2^3\right)x\left(2^3\right)x\left(2^3\right)x\left(2^3\right)x\left(2^3\right) = 2^{3+3+3+3+3} = 2^{3x5} = 2^{15}$$

So $\left(2^3\right)^5 = 2^{15}$

Exercise 16. Fill in the missing numbers.

a) $\left(4^2\right)^5 = 4^{[\ \ ]}$ **b)** $\left(3^2\right)^{[\ \ ]} = 3^8$ **c)** $\left(3^{[\ \ ]}\right)^2 = 3^8$

d) $\left([\ \ ]^2\right)^3 = 5^6$ **e)** $\left(2^2\right)^{[\ \ ]} = [\ \ ]^8$

2.4 Powers with different base but the same exponent

Multiplication: When powers **with the same exponent** are multiplied, multiply the bases and keep the **same exponent.**

Example:

$$2^5 x 7^5 = (2x7)^5 = 14^5$$

Division: When powers **with the same exponent** are divided, bases are divided and the **exponent remains unchanged.**

Example:

$8^3 : 2^3 = (8:2)^3 = 4^3$, we can also use fractions notation $\frac{8^3}{2^3} = \left(\frac{8}{2}\right)^3 = 4^3$

Exercise 17. Fill in the missing numbers.

a) $3^7 \cdot 8^7 = [\quad]^7$ **b)** $[\quad]^2 \cdot [\quad]^2 = 6^2$ **c)** $5^2 \cdot [\quad]^2 = 15^2$

d) $2^2 \cdot [\quad]^2 = 14^{[\quad]}$ **e)** $8^5 : 4^5 = [\quad]^5$ **f)** $\frac{15^5}{5^5} = [\quad]^5$

g) $\frac{16^7}{8^7} = [\quad]^7$ **h)** $\frac{6^{12}}{3^{12}} = [\quad]^{[\quad]}$

Exercise 18. Operate.

a) $7^3 \cdot 7^5 = 7^{[\quad]}$ **b)** $4^2 \cdot 4^3 \cdot 4^6 \cdot 4 =$ **c)** $9^2 \cdot 9^7 \cdot 9^2 =$

d) $5^7 : 5^3 =$ **e)** $(2^2 \cdot 2^6) : 2^3 =$ **f)** $(14^2)^4 =$

g) $3^7 : (3^2 \cdot 3^3) =$ **h)** $(2^2 \cdot 3^2) = [\quad]^2$ **i)** $(12^2 \cdot 12^3)^4 =$

j) $(6^2 : 3^2) = [\quad]^2$ **k)** $[3^8 : (3^2 \cdot 3^3)]^2 = 3^{[\quad]}$ **l)** $\frac{5^7 \cdot 5^3}{5^4} =$

3 Square roots.

The inverse operation of power is root.

The inverse of a square is a square root, that is: If we say that $\sqrt{9} = 3$, that means that $3^2 = 9$

10. Calculate the following square roots:

a) $\sqrt{81} =$ **b)** $\sqrt{121} =$ **c)** $\sqrt{100} =$ **d)** $\sqrt{10000} =$

e) $\sqrt{900} =$ **f)** $\sqrt{1600} =$ **g)** $\sqrt{250000} =$

There are numbers that are not squares of any other number, for example between 9 and 16 (squares of 3 and 4); there is not any whole square number.

The square root of all numbers between 9 and 16 are between 3 and 4, for example

$3 < \sqrt{13} < 4$, this is an estimation of the $\sqrt{13}$ value.

Exercise 19. Estimate the value of the following square roots:

a) $< \sqrt{57} <$ **b)** $< \sqrt{250} <$ **c)** $< \sqrt{700} <$

d) $< \sqrt{1500} <$ **e)** $< \sqrt{30} <$ **d)** $< \sqrt{2057} <$

When we estimate a square root as $3 < \sqrt{13} < 4$ we can also say that $\sqrt{13}$ is 3 and the difference of 13 and 9 (square of 3), which is 4, is called the **remainder.**

So we say that the **square root of 13 is 9 and the remainder is 4**.

That means $13 = 3^2 + 4$

Exercise 20. Calculate the square roots and the remainders for the numbers of the previous exercise; write the meanings as in the example.

3 Multiples and factors

1 Multiples

The products of a number with the **natural numbers** 1, 2, 3, 4, 5, ... are called the **multiples** of the number.

For example:

$$7 \times 1 = 7$$
$$7 \times 2 = 14$$
$$7 \times 3 = 21$$
$$7 \times 4 = 28$$

So, the multiples of 7 are: 7, 14, 21, 28, and so on.

Note:
The multiples of a number are obtained by multiplying the number by each of the natural numbers.

For example:

- multiples of 2 are 2, 4, 6, 8, …
- multiples of 3 are 3, 6, 9, 12, …
- multiples of 4 are 4, 8, 12, 16, …

Example 1

Write down the first ten multiples of 5.
Solution:
The first ten multiples of 5 are 5, 10 15, 20, 25, 30, 35, 40, 45, 50.

Exercise 1

a) Write down all the multiples of 6 between 20 and 70

b) Write down all the multiples of 7 between 30 and 80

c) Write the three smallest multiples of 8 which are over 50

d) **Write the smallest multiple of 37 which is over 500**

2 Factors

A whole number that divides exactly into another whole number is called a **factor** of that number.

For example $20:4=5$

So, 4 is a factor of 20 as it divides exactly into 20.

We could also consider than 20:5 = 4

So, 5 is a factor of 20 as it divides exactly into 20.

Note:
If a number can be expressed as a <u>product</u> of two whole numbers, then the whole numbers are called **factors** of that number.

For example $20=1\,x\,20=2\,x\,10=4\,x\,5$

So, the factors of 20 are 1, 2, 4, 5, 10 and 20.

Example 2

List all the factors of 42.

Solution: $42=1x42=2x21=3x14=6x7$

So, the factors of 42 are 1, 2, 3, 6, 7, 14, 21 and 42.

Example 3

Is 7 a factor of 15?
Solution:
$$15\div 7=\begin{cases}\text{quotient } 2\\ \text{remainder } 1\end{cases}.$$
Clearly 7 does not divide exactly into 15, so 7 is not a factor of 15

Exercise 2
Write down all the factors of

a) 60

b) 20

c) 100

3 Prime Numbers

If a number has only two different <u>factors</u>, 1 and itself, then the number is said to be a **prime number.**

For example, 7 = 1x7

7 is a prime number since it has only two different factors.

2 = 1x2, 3 = 1x3, 5 = 1x5,… 2,3,5,…. Are prime numbers

Exercise 3

The Sieve of Eratosthenes

A Greek mathematician, Eratosthenes (276-195 BC), discovered the Sieve which is known as the Sieve of Eratosthenes, it is a method to get prime numbers.

3.1 We start with a table of whole numbers e.g. from 1 to 200 and cross out the number 1, as it has been done below.

~~1~~	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200

3.2. Circle the number 2 and then cross out all the multiples of 2, as shown below.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	9	~~10~~
11	~~12~~	13	~~14~~	15	~~16~~	17	~~18~~	19	~~20~~
21	~~22~~	23	~~24~~	25	~~26~~	27	~~28~~	29	~~30~~
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
181	~~182~~	183	~~184~~	185	~~186~~	187	~~188~~	189	~~190~~
191	~~192~~	193	~~194~~	195	~~196~~	197	~~198~~	199	~~200~~

3.3. The next number that is not crossed out is 3. Circle it and then cross out all the multiples of 3: 3, 6, 9, 12....

3.4. The next number that is not crossed out is 5. Circle it and then cross out all the multiples of 5: 5, 10, 15, 20….

3.5. The next number that is not crossed out is 7. Circle it and then cross out all the multiples of 7.

3.6. Continue this process until there is no number to be crossed.

3.7. Make a list of all the circled numbers.

3.8. Write the factors of each of the circled numbers.

3.9. Make a list of the first twenty crossed out numbers and write the factors of these numbers.

3.10 What do you observe about the number of factors of the circled numbers and the crossed out numbers? Write a brief sentence in your own words.

3.11. What name is given to the circled numbers?

3.12. What name is given to the crossed out numbers?

3.13. How many prime numbers are less than 100?

4 Tests of divisibility

One number is divisible by:

2 If the last digit is 0 or is divisible by 2, (0, 2, 4, 8).

3 If the sum of the digits is divisible by 3.

4 If the last two digits are divisible by 4.

5 If the last digit is 0 or is divisible by 5, (0,5).

9 If the sum of the digits is divisible by 9.

8 If the half of it is divisible by 4.

6 If it is divisible by 2 and 3.

11 If the sum of the digits in the even position minus the sum of the digits in the uneven position is 0 or divisible by 11.

Exercise 4 Find which of the numbers: 239 300 675 570 800 495 888 6402 2088 are multiples of:

a) 3 →

b) 2 →

c) 5 →

d) 4 →

e) 11 →

f) 9 →

Exercise 5 Find the factor decomposition of the following numbers:

24 | 123 | 420 | 4752 |

24 = 123 = 420 = 4752 =

5 Common Multiples

<u>Multiples</u> that are common to two or more numbers are said to be **common multiples**.

E.g. Multiples of 2 are 2, 4, **6**, 8, 10, **12**, 14, 16, **18**, …
Multiples of 3 are 3, **6**, 9, **12**, 15, **18**, …

So, common multiples of 2 and 3 are 6, 12, 18, …

Example 4

Find the common multiples of 4 and 6.
Solution:
Multiples of 4 are 4, 8, **12**, 16, 20, **24**, 28, 32, **36**, …
Multiples of 6 are 6, **12**, 18, **24**, 30, **36**, …
So, the common multiples of 4 and 6 are 12, 24, 36, …

Exercise 6

a) Find the sequence of the common multiples of 3 and 5.

b) Find the sequence of the common multiples of 12 and 9.

Lowest common multiple

The smallest common multiple of two or more numbers is called the **lowest common multiple** (LCM).

E.g. Multiples of 8 are 8, 16, **24**, 32, …
Multiples of 3 are 3, 6, 9, 12, 15, 18, 21, **24**, …

LCM of 3 and 8 is 24

Method I (for small numbers)

To find the lowest common multiple (LCM) of two or more numbers, list the multiples of the largest number and stop when you find a multiple of the other number. This is the LCM.

Example 5

Find the lowest common multiple of 6 and 9.
Solution:
List the multiples of 9 and stop when you find a multiple of 6.

Multiples of 9 are 9, **18**, …
Multiples of 6 are 6, 12, **18**, …
LCM of 6 and 9 is 18

Example 6

Find the lowest common multiple of 5, 6 and 8.
Solution:
List the multiples of 8 and stop when you find a multiple of both 5 and 6.
Multiples of 8 are 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96, 104, 112, 120, …
Stop at 120 as it is a multiple of both 5 and 6.
So, the LCM of 5, 6 and 8 is 120.

Exercise 7 Find the LCM of

a) **6 and 8**

b) **10 and 20**

c) **8 and 12**

Method II (General)

To find the lowest common multiple (LCM) of higher numbers:

- Find the prime factor decomposition.

- Choose the **non common factors** and the **common factors with the highest exponents.**

Example 7

Find the lowest common multiple of 18 and 24.
Solution:

$18 = 2 \cdot 3^2$
$24 = 2^3 \cdot 3$ So, the LCM of 18 and 24 is $LCM = 2^3 \cdot 3^2 = 72$.

Exercise 8 Find the LCM of

a) 150 and 350

b) 100 and 120

c) 120, 480 and 180

6 Common Factors

Factors that are common to two or more numbers are said to be **common factors**.

For example $4 = 1x4 = 2x2$
$6 = 1x6 = 2x3$

- Factors of 4 are 1,2 and 4
- Factors of 6 are 1, 2, 3 and 6

So, the common factors of 4 and 6 are 1 and 2

Example 8

Find the common factors of 10 and 30.
Solution:
$10 = 1x10 = 2x5$
$30 = 1x30 = 2x15 = 5x6 = 3x10$

So, the common factors of 10 and 30 are **1, 2, 5** and **10**.

Example 9

Find the common factors of 26 and 39.
Solution:
$26 = 1x26 = 2x13$

$39 = 1x39 = 3x13$
So, the common factors of 26 and 39 are **1** and **13**.

7 Highest Common Factor

The largest common factor of two or more numbers is called the **highest common factor** (HCF).

For example $8 = 1x8 = 2x4$
$12 = 1x12 = 2x6 = 3x4$

- Factors of 8 are 1,2, 4 and 8
- Factors of 12 are 1, 2, 3, 4, 6 and 12

So, the common factors of 8 and 12 are 1, 2 and 4 HCF is **4**

Example 10

Find the highest common factor of 14 and 28.
Solution:
$14 = 1x14 = 2x7$

$28 = 1x28 = 2x14 = 4x7$

HCF = **14**

To find the **Highest Common Factor** of higher numbers:
- Find the prime factor decomposition.
- Choose only the **common factors with the lowest exponents.**

Exercise 9 Find the HCF and the LCM of:

a) 18 and 24

b) 180 and 40

c) 60, 320 and 140

EXERCISES

1. Which numbers between 37 and 74 have a factor of 3?

2. Which of these is a multiple of 6?

122, 28, 30, 402, 634, 348, 10,500

3. List all the prime numbers between 20 and 50.

4. List all the factors of:

a) 12 b) 30 c) 66 d) 200

5. In a bus station there is a bus leaving for London every 45 minutes and one leaving for Brighton every 60 minutes. If a bus to London and a bus to Brighton leave at the same time, how many minutes will it be before two buses leave again at the same time?.

6. Find the prime factorization of both 156 and 250. What is the HCF of these numbers? What is the LCM?

7. List all the common factors of 30 and 75. What is the HCF of 30 and 75? What is the LCM of 30 and 75?

8. Find the HCF and the LCM of 36 and 90.

9. Lara has a day off every six days and Dave has a day off every eight days, if they both have a day off on the first of November, which day will they have the same day off again?

10. What numbers that are less than 100 are multiples of 3 and 5?

11. How many different rectangles with an area of 36 cm^2 using only whole numbers (centimetres), can be made?

12. Three traffic lights are placed along the same avenue at three different crosses. The first one changes every 20 seconds, the second, every 30 seconds and the third every 28 seconds. They have changed to green simultaneously. How long does it take until they change again at the same time? Explain your answer.

13. Marta has 12 red, 30 green and 42 yellow marbles and she wants to put them in boxes, as many as possible, all the boxes with the same amount of each colour and with no marbles remaining. How many boxes will she have? How many marbles of each colour are there in each box?

4 Fractions

1 Fractions

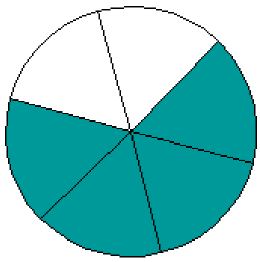
A fraction is a number that expresses part of a unit or a part of a quantity.

Fractions are written in the form $\frac{a}{b}$ where a and b are whole numbers, and the number b is not 0.

They can be written also in the form a/b

The number *a* is called the **numerator**, and the number *b* is called the **denominator**.
The numerator tells us how many equal parts we have.
The denominator tells us how many equal parts are available.

The fraction $\frac{4}{6}$ represents the shaded portion of the circle on the right. There are 6 pieces in the group, and 4 of them are shaded.



2 Reading fractions

We use the cardinals to name the numerator and the ordinals for the denominator with two exceptions when the denominators are 2 and 4, for denominator larger than 10 we say "over" and do not use ordinal, so we read:

$\frac{1}{2}$ one half $\frac{3}{2}$ three halves $\frac{2}{3}$ two thirds

$\frac{1}{4}$ a quarter or a fourth $\frac{12}{15}$ twelve over fifteen

$\frac{3}{4}$ three quarters or three fourths $\frac{6}{10}$ six tenths

$\frac{17}{32}$ seventeen over thirty-two $\frac{7}{5}$ seven fifths……

Exercise 1 Write in words the following fractions

$\frac{11}{4} \Rightarrow$

$\frac{3}{8} \Rightarrow$

$\frac{9}{6} \Rightarrow$

$\frac{17}{2} \Rightarrow$

$\frac{4}{9} \Rightarrow$

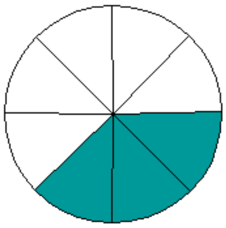
$\frac{19}{62} \Rightarrow$

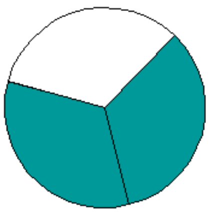
$\frac{1}{12} \Rightarrow$

$\frac{14}{93} \Rightarrow$

$\frac{17}{903} \Rightarrow$

Exercise 2 Write and read the fractions that represent the shaded portions.





3 Equivalent fractions

Equivalent fractions are different fractions that name the same amount.
Examples:
- 1/2 is equivalent to 2/4, because we have multiplied both the numerator and the denominator by 2 and the total amount does not change.

- 12/20 is equivalent to 3/5, because we have divided both the numerator and the denominator by 4.

- The fractions 1/2, 2/4, 3/6, 100/200, and 521/1042 are all equivalent fractions.

- The fractions 3/5, 6/10, 12/20 and 24/40 are all equivalent fractions.

Exercise 3 Write a sequence of equivalent fractions as in the example:

Starting fraction	Equivalent fractions					
$\frac{1}{3}$	$\frac{2}{6}$	$\frac{3}{9}$	$\frac{4}{12}$	$\frac{5}{15}$	$\frac{6}{18}$	$\frac{7}{21}$
$\frac{2}{5}$						
$\frac{8}{16}$						
$\frac{16}{32}$						
$\frac{7}{3}$						
$\frac{18}{27}$						

We can test if two fractions are equivalent by cross-multiplying their numerators and denominators. This is also called taking the cross-product.

$$\frac{a}{b} \qquad \frac{c}{d}$$

So if we want to test if $\frac{12}{20}$ and $\frac{24}{40}$ are equivalent fractions

The first cross-product is the product of the first numerator and the second denominator: $12 \times 40 = 480$.

The second cross-product is the product of the second numerator and the first denominator: $24 \times 20 = 480$.
Since the cross-products are the same, the fractions are equivalent.

Exercise 4

a) Test if 3/7 and 18/42 are equivalent fractions.

b) Test if 2/4 and 13/20 are equivalent fractions.

Simplest form

We know that 4/12 = 2/6 = 1/3
4 and 12 have a common factor (4), so 4/12 can be written as 1/3 (Divide the top and the bottom by 4.)

2 and 6 have a common factor (2), so 2/6 can be written as 1/3 (Divide the top and the bottom by 2.)

However, 1 and 3 have no common factors, so **1/3 is the simplest form of these fractions.**

There are two methods of reducing a fraction to the lowest terms.

Method 1:
Divide the numerator and denominator by their HCF.
12/30 = (12 : 6)/(30 : 6) = 2/5

Method 2:
Divide the numerator and denominator by any common factor. Keep dividing until there are no more common factors.

12/30 = (12 : 2)/(30 : 2) = 6/15 = (6 : 3)/(15 : 3) = 2/5

Exercise 5 Express these fractions in the simplest form. (Some may already be in the simplest form)

a) $\frac{5}{10}$ b) $\frac{3}{9}$ c) $\frac{7}{21}$ d) $\frac{9}{51}$

e) $\frac{48}{84}$ f) $\frac{17}{51}$ g) $\frac{9}{72}$ h) $\frac{64}{88}$

i) $\frac{25}{125}$ j) $\frac{30}{100}$ k) $\frac{21}{49}$ l) $\frac{52}{90}$

Exercise 6 Express, in the simplest form, which fraction corresponds to these situations:

a) **In a bag of 90 pens, 15 are blue**

b) **The number of girls and boys in our class**

c) **There are 90 pupils of the 270 who come by bus to the school.**

4 Comparing and ordering fractions

1. To compare fractions with the same denominator, look at their numerators. The largest fraction is the one with the largest numerator.

2. To compare fractions with different denominators, take the cross product. Compare the cross products.

   a. If the cross-products are equal, the fractions are equivalent.

   b. If the first cross product is larger, the first fraction is larger.

   c. If the second cross product is larger, the second fraction is larger.

Examples: Compare the fractions 3/7 and 1/2.

The first cross-product is: $3 \times 2 = 6$.
The second cross-product is: $7 \times 1 = 7$.
Since the second cross-product is larger, the second fraction is larger.

Compare the fractions 13/20 and 3/5.

The first cross-product is: $5 \times 13 = 65$.
The second cross-product is: $20 \times 3 = 60$.
Since the first cross-product is larger, the first fraction is larger.

Exercise 7 Order the following fractions:

a) $\frac{3}{7}, \frac{2}{5}$

b) $\frac{8}{13}, \frac{3}{5}$

c) $\frac{3}{4}, \frac{8}{11}, \frac{22}{19}$

d) $\frac{7}{12}, \frac{18}{7}, \frac{9}{17}$

5 Adding and subtracting fractions

1. If the fractions have the same denominator:

The numerator of the sum is found by simply adding the numerators over the denominator.

Their difference is the difference of the numerators over the denominator.

We do not add or subtract the denominators!

Reduce if necessary.

Examples: 3/8 + 2/8 = 5/8   9/2 - 5/2 = 4/2 = 2

2. If the fractions have different denominators

For example $\frac{3}{5}+\frac{7}{8}$

1) First, find the lowest common denominator.

For this problem complete the following steps:

a) We find the LCM of the denominators as it is the smallest number into which both denominators will divide. For 5 and 8, it is easy to see that 40 is the LCM. We look for equivalents fractions with 40 as common denominator. $\frac{3}{5}=\frac{[\ ]}{40}$ and $\frac{7}{8}=\frac{[\ ]}{40}$

b) We divide every new denominator by the previous one and we multiply the result by the numerator.

$\frac{3}{5}=\frac{[\ ]}{40}$ We divide the new denominator 40 by the previous one 5 what gives us 8, we must multiply 3 by 8, so $\frac{3}{5}=\frac{24}{40}$.
With the same process:

$\frac{7}{8}=\frac{[\ ]}{40}=\frac{35}{40}$ We have multiplied top and bottom by 5

2) Add the numerators and do not change the denominator.

$$\frac{3}{5}+\frac{7}{8}=\frac{24}{40}+\frac{35}{40}=\frac{24+35}{40}=\frac{59}{40}$$

3) Reduce if necessary.

Exercise 8 Find and reduce:

a) $\frac{3}{4}+\frac{5}{4}$ b) $\frac{3}{13}+\frac{5}{13}$

c) $\frac{1}{4}+\frac{3}{4}$ d) $\frac{7}{5}-\frac{3}{5}$

e) $\frac{6}{7}-\frac{3}{7}+\frac{2}{7}$

f) $\frac{1}{2}-\frac{3}{2}$

g) $\frac{4}{5}+\frac{3}{10}$

h) $\frac{3}{8}+\frac{3}{16}$

i) $\frac{1}{6}+\frac{2}{15}$

j) $\frac{5}{12}-\frac{3}{16}$

k) $\frac{4}{3}+\frac{3}{4}$

l) $\frac{1}{2}+\frac{3}{4}+\frac{7}{6}$

m) $\frac{3}{4}-\frac{1}{2}+\frac{3}{8}$

n) $\frac{11}{14}-\frac{1}{7}+\frac{3}{2}$

o) $\frac{7}{12}-\frac{13}{16}+\frac{8}{5}$

Exercise 9 Laura bought a 1/2 kg bag of sweets. She ate 1/3 kg herself and gave the rest to Robert. How much did she give to Robert?

6 Improper fractions Mixed numbers

Improper fractions have numerators that are larger than or equal to their denominators.

For example $\frac{15}{7}$, $\frac{7}{7}$, and $\frac{18}{3}$ are improper fractions.

Mixed numbers have a whole number part and a fraction part.

For example $2\frac{3}{5}$, $5\frac{1}{3}$ are mixed numbers, meaning $2\frac{3}{5}=2+\frac{3}{5}$ and

$5\frac{1}{3}=5+\frac{1}{3}$.

Converting improper fractions to mixed numbers

To change an improper fraction into a mixed number, divide the numerator by the denominator. The quotient is the whole part and the remainder is the numerator of the fractional part.

For example

$$\frac{15}{4},\ 15:4 \Rightarrow \begin{cases} \text{quotient } 3 \\ \text{remainder } 3 \end{cases} \text{so } \frac{15}{4}=3\frac{3}{4}$$

$$\frac{17}{3},\ 17:3 \Rightarrow \begin{cases} \text{quotient } 5 \\ \text{remainder } 2 \end{cases} \text{so } \frac{17}{3}=5\frac{2}{3}$$

Converting mixed numbers to improper fractions

To change a mixed number into an improper fraction, multiply the whole number by the denominator and add it to the numerator of the fractional part.

For example

$$2\frac{3}{5}=2+\frac{3}{5}=\frac{2\cdot 5+3}{5}=\frac{13}{5}$$

$$7\frac{2}{3}=\frac{7\cdot 3+2}{3}=\frac{23}{3}$$

Note that converting mixed numbers to improper fractions is the same as adding whole numbers and fractions

Exercise 10 Operate converting first to improper fractions and express the result as a mixed number.

a) $1\frac{3}{2}+\frac{1}{4}+3\frac{7}{8}$

b) $3\frac{1}{2}+4\frac{1}{3}$

c) $2\frac{1}{4}-1\frac{2}{5}$

d) $3\frac{1}{6}+2\frac{1}{4}-1\frac{1}{3}$

7 Multiplying fractions

When two fractions are multiplied, the result is a fraction with a numerator that is the product of the fractions' numerators and a denominator that is the product of the fractions' denominators. **Reduce when possible.**

Examples:

$\frac{3}{8}\cdot\frac{3}{5}=\frac{3\cdot 3}{8\cdot 5}=\frac{9}{40}$ i) $\frac{7}{6}\cdot\frac{2}{5}=\frac{7\cdot 2}{6\cdot 5}=\frac{7}{3\cdot 5}=\frac{7}{15}$ We cancel the common factor of 2 in the top and bottom of the product. Remember that like factors in the numerator and denominator cancel out.

Exercise 11 Multiply giving your answers in the simplest form

a) $\frac{4}{5}\cdot\frac{7}{11}=$

b) $\frac{14}{15}\cdot\frac{15}{19}=$

c) $\frac{5}{4}\cdot\frac{2}{7}=$

d) $\frac{5}{6}\cdot\frac{2}{19}=$

e) $\frac{1}{3}\cdot\frac{5}{7}\cdot\frac{17}{2}=$

f) $\frac{5}{9}\cdot\frac{2}{5}\cdot\frac{3}{7}=$

g) $\frac{12}{13}\cdot\frac{26}{21}\cdot\frac{7}{23}=$

h) $\frac{120}{7}\cdot\frac{3}{40}\cdot\frac{14}{9}=$

8 Multiplying a fraction by a whole number Calculating a fraction of a quantity

To multiply a fraction by a whole number, choose one of the two methods:

1. Write the whole number as an improper fraction with a denominator of 1, and then multiply as fractions.

2. Multiply the whole number by the numerator and do not change the denominator.

Examples:

a) $6\cdot\frac{2}{7}=\frac{6}{1}\cdot\frac{2}{7}=\frac{12}{7}$ b) $5\cdot\frac{3}{19}=\frac{5\cdot 3}{19}=\frac{15}{19}$

To calculate a fraction of a quantity which is multiple of the denominator it is more convenient to divide it by the denominator and then multiply by the numerator.

Example:

$\frac{5}{8}$ of 192 kg.

$192:8=32$, and $32x5=160$. So the $\frac{5}{8}$ of 192 kg are 160 kg

Exercise 12 Operate giving your answers in the simplest form

a) $42\cdot\frac{5}{6}=$ b) $198\cdot\frac{4}{9}=$

c) $\frac{5}{4}\cdot 37=$ d) $\frac{5}{8}\cdot 18=$

e) $2\cdot\frac{3}{9}\cdot\frac{17}{5}=$ f) $\frac{2}{8}\cdot 3\cdot\frac{3}{7}=$

g) $7\cdot\frac{66}{21}\cdot\frac{17}{13}=$ h) $15\cdot\frac{3}{4}=$

Exercise 13 Calculate:

a) $\frac{5}{6}$ of 2 days (in hours) b) $\frac{3}{5}$ of a litre (in ml)

c) $\frac{3}{7}$ of 1470 g

d) $\frac{1}{3}$ of 30 pupils

e) $\frac{7}{9}$ of 270 ml

f) $\frac{2}{21}$ of 9 weeks (in days)

g) $\frac{3}{8}$ of 3000 €

h) $\frac{3}{4}$ of 7 tonnes (in kg)

Exercises 14

14.1 There are 720 pupils in a school, find the number of pupils in each group if:

a) $\frac{2}{3}$ **enjoy mathematics**

b) $\frac{5}{6}$ **enjoy sports**

c) $\frac{3}{8}$ **do not do their homework**

d) $\frac{9}{20}$ **expect to go to university**

14.2 There are 360º in one complete revolution. Calculate the degrees of the following angles:

a) $\frac{3}{5}$ **of a revolution**

b) $\frac{7}{12}$ **of a revolution**

c) $\frac{5}{6}$ **of a revolution**

d) $\frac{5}{9}$ **of a revolution**

e) $\frac{3}{4}$ **of a revolution**

9 Dividing fractions

To divide fractions, multiply the first one by the reciprocal of the second fraction.

The reciprocal of a fraction is obtained by switching its numerator and denominator.

We can also take the cross product.

To divide a number by a fraction, multiply the number by the reciprocal of the fraction.

Example:

$\frac{4}{5}:\frac{7}{11}=\frac{4}{5}\cdot\frac{11}{7}=\frac{44}{35}$ or simply taking the cross product $\frac{4}{5}:\frac{7}{11}=\frac{4x11}{5x7}=\frac{44}{35}$

Exercise 15 Operate:

a) $\frac{3}{2}:\frac{4}{9}=$

b) $\frac{1}{5}:\frac{5}{19}=$

c) $\frac{3}{14}:\frac{4}{7}=$

d) $\frac{15}{3}:\frac{5}{6}=$

e) $\left(\frac{5}{3}:\frac{5}{8}\right):\frac{7}{4}=$

f) $2:\left(\frac{9}{7}:\frac{13}{3}\right)=$

g) $\left(2 : \frac{9}{7}\right) : \frac{13}{3} =$

h) $\left[\left(\frac{20}{55} : 5\right) : \frac{3}{40}\right] \cdot \frac{14}{9} =$

Exercises 16

16.1. In a factory a metal rod 49 metre long is divided into $\frac{7}{5}$ metre pieces. How many pieces can be made from each rod?

16.2. A math's teacher spends $\frac{4}{5}$ of his working time in school teaching his classes. If $\frac{2}{9}$ of this teaching time is spent with his first year classes. What fraction of his working time does he spend teaching the first year?

16.3 A seven tonne load of soil container is divided in $\frac{3}{140}$ tonne bags. How many bags can be filled?

16.4. How many pieces would there be if $4\frac{3}{10}$ metres of ribbon were divided into tenths?

16.5 Last night John spent $\frac{3}{4}$ of an hour on English homework and $\frac{5}{6}$ of this time, on his maths. If he started his homework at 20 o'clock, did he finish it in time to watch The Simpsons, which started at 21 o'clock? Explain your answer.

16.6 Dolly opened a $\frac{3}{4}$ litre bottle of coke, James drank $\frac{1}{3}$ litre and Carole $\frac{1}{5}$ litre. How much coke was left for Dolly?

5 Decimal numbers

1 Decimal numbers

Decimal numbers such as 3.762 are used in situations in which we look for more precision than whole numbers provide.

As with whole numbers, a digit in a decimal number has a value which depends on the place of the digit. The places to the left of the decimal point are ones, tens, hundreds, and so on, just as with whole numbers. This table shows the decimal place value for various positions:

Note that adding extra zeros to the right of the last decimal digit does not change the value of the decimal number.

Place (underlined)	Name of Position
$\underline{1}.234567$	Ones (units) position
$1.\underline{2}34567$	Tenths
$1.2\underline{3}4567$	Hundredths
$1.23\underline{4}567$	Thousandths
$1.234\underline{5}67$	Ten thousandths
$1.2345\underline{6}7$	Hundred Thousandths
$1.23456\underline{7}$	Millionths

Example:

In the number 3.762, the 3 is in the units place, the 7 is in the tenths place, the 6 is in the hundredths place, and the 2 is in the thousandths place.

- 3 is called the whole number portion
- 762 is the decimal portion

2 How to read decimal numbers.

We have to read the whole number, then the word "point" and the decimal numbers one by one.

Example:

In the number 2.34 we say two point three four

We read the number 0.057 as nought point zero five seven.

Exercise 1

Write in words the following:

21.456

0.77

0.0089

5.7254

3 Adding and subtracting decimals

To add or subtract decimals, line up the decimal points and then follow the rules for adding or subtracting whole numbers, placing the decimal point in the same column as above.

When one number has more decimal places than another, use 0's to give them the same number of decimal places.

Example:

Add 43.67 + 2.3

1) Line up the decimal points and adds a 0 on the right of the second:

2) Then add.

$$\begin{array}{r} 43.67 \\ \underline{\ 2.30} \\ 45.97 \end{array}$$

Example:

Subtract. 57.8 – 8.06

1) Line up the decimal points.

2) Add extra 0's

3) Subtract.

$$\begin{array}{r} 57.80 \\ \underline{\ 8.06} \\ 49.74 \end{array}$$

4 Multiplying decimal numbers

Multiplying decimals is just like multiplying whole numbers. The only extra step is to decide how many digits to leave to the right of the decimal point. To do that, add the numbers of digits to the right of the decimal point in both factors.

Example:

Multiply 23.56 x 34.1

$$\begin{array}{r} 23.56 \\ 34.1 \\ \hline 2356 \\ 9424 \\ 7068 \\ \hline 803.396 \end{array}$$

Exercise 2 Calculate

a) 5.6 x 6.9

b) 12.37 x 76.78

c) –4.66 x 4.7

d) 0.345 (32.4 – 4.67)

5 Dividing whole numbers, with decimals

Continue the whole division adding zeros to the right of the number being divided until you get the amount of decimal digits required.

Example:

Divide 235:6 until the hundredth

```
235  | 6
      -------
  55   39.16
   10
    40
     4
```

Exercise 3 Calculate with two decimal digits

a) 56 : 7 b) 7634 : 34

c) – 679 : 32 d) 9783 : 127

6 Dividing decimals by decimals

To divide by a decimal, multiply that decimal by a power of 10 great enough to obtain a whole number. Multiply the dividend by that same power of 10. Then the problem becomes one involving division by a whole number instead of division by a decimal.

Exercise 4 Calculate with three decimal digits

a) 56.7 : 2.34 b) 1432.3 : 0.42

c) – 12.34 : 3.5 d) 1 : 1.2

7 Rounding Decimal Numbers

To round a number to any decimal place value, we want to find the number with zeros in all of the lower places that is closest in value to the original number. As with whole numbers, we look at the digit to the right of the place we wish to round to.

Note: When the digit 5, 6, 7, 8, or 9 appears in the ones place, round up; when the digit 0, 1, 2, 3, or 4 appears in the ones place, round down.

Exercise 5 Round:

1.17 to the nearest tenth

2.375 to the nearest hundredth

0.7084 to the nearest thousandth

12.87 to the nearest unit

151.504 to the nearest hundred

7478 to the nearest thousand

8 Writing a fraction as a decimal

Method 1 - Convert to an equivalent fraction whose denominator is a power of 10, such as 10, 100, 1000, 10000, and so on, then write in decimal form.

Examples:

$1/4 = (1 \times 25)/(4 \times 25) = 25/100 = 0.25$

$3/20 = (3 \times 5)/(20 \times 5) = 15/100 = 0.15$

Method 2 - Divide the numerator by the denominator. Round to the decimal place asked for, if necessary.

Examples:

$13/4 = 13 \div 4 = 3.25$

Convert 3/7 to a decimal number. Round it to the nearest thousandth.

We divide one decimal place past the place we need to round to, then round the result.

$3/7 = 3 \div 7 = 0.4285\ldots$

That equals 0.429 when rounded to the nearest thousandth.

Exercise 6 Convert to a decimal. Round it to the nearest hundredth.

a) $\frac{4}{3}$ b) $\frac{13}{6}$

c) $\frac{11}{7}$

d) $\frac{7}{4}$

9 Repeating decimals

Every fraction can be written as a decimal.

For example, 1/3 is 1 divided by 3.

If you use a calculator to find 1 ÷ 3, the calculator returns 0.333333... This is called a **repeating decimal**. To represent the idea that the 3's repeat forever, one uses an arc

$$\frac{1}{3} = 0.33333.... = 0.\overset{\frown}{3}$$

In Britain they use a horizontal bar $\frac{1}{3} = 0.33333.... = 0.\overline{3}$

0 is the whole number portion

3333… is the decimal portion

3 is called the period or the recurring number, there is a period of one digit

Another example

$$\frac{13}{11} = 1.181818.... = 1.\overset{\frown}{18} \text{ or } \frac{13}{11} = 1.181818.... = 1.\overline{18}$$

1 is the whole number portion

1818… is the portion

18 is the period, this period has two digits

If there is no mark over the number it means that it has been an exact division. These numbers are called **regular numbers.**

Example 2/5 = 0.4 is a regular number

There can be decimals numbers without repeating decimals

Examples: 1.01001000100001……

$\pi = 3.14159........$

These are called irrational numbers and can not be written as fractions.

Exercise 7 Write as a decimal:

a) $\frac{17}{6}$ b) $\frac{13}{7}$

c) $\frac{131}{11}$ d) $\frac{71}{9}$

Exercises

1 Ellen earns £137.40 per week and after 4 weeks she gets an extra payment of £24.75, she spends £354.60 in this period. How much has she saved?

2 A student has been studying a total time of 4 h 35 min and during this time has been writing for 100 min. How long, in hours has he been studying without writing?

3 Susan cooked a cake and used 1.35kg of flour, 0.37kg of sugar, 3 eggs that weigh 80g each and 240g of milk. Which is the weight of the mixture?

4 I buy 7 mugs and pay 53.55 €. How much does each mug cost?

5 Henry had 83.40 €. He bought four tickets for the cinema at 6.50 each and 2 bags of pop corn at 2.25 each. How much money has he got left?

6 A breeder gives to each pig 0.65 kg of food for every 4 kg of body weight. There are 4 pigs of 75.8 kg, 56.4 kg, 75.4 kg and 89.3 kg. How much food must be prepared in total?

7 In a restaurant 7 friends are having a meal, the bill is £173.6 and each person contributes £25.50. What tip does the waiter receive?

8 A company produces items that are sold for £13.63 each, the daily production is 1275 items and the cost of production is £11,324.50. What is the daily income for the company?

9 Three partners share a benefit of £18538, the first one receives a third minus £173, the second one £5938.55 £. How much does the last one receive?

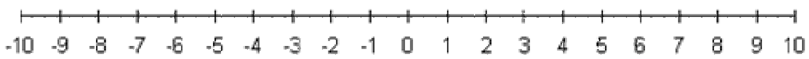
6 Integers

1 The negative numbers.

There are many situations in which you need to use numbers below zero, one of these is temperature, others are money that you can deposit (positive) or withdraw (negative) in a bank, steps that you can take forwards (positive) or backwards (negative).

Positive integers are all the whole numbers greater than zero: 1, 2, 3, 4, 5, ... Negative integers are all the opposites of these whole numbers: -1, -2, -3, -4, -5, ….

The Number Line
The number line is a line labelled with the integers in increasing order from left to right, that extends in both directions:



For any two different places on the number line, the integer on the right is greater than the integer on the left.

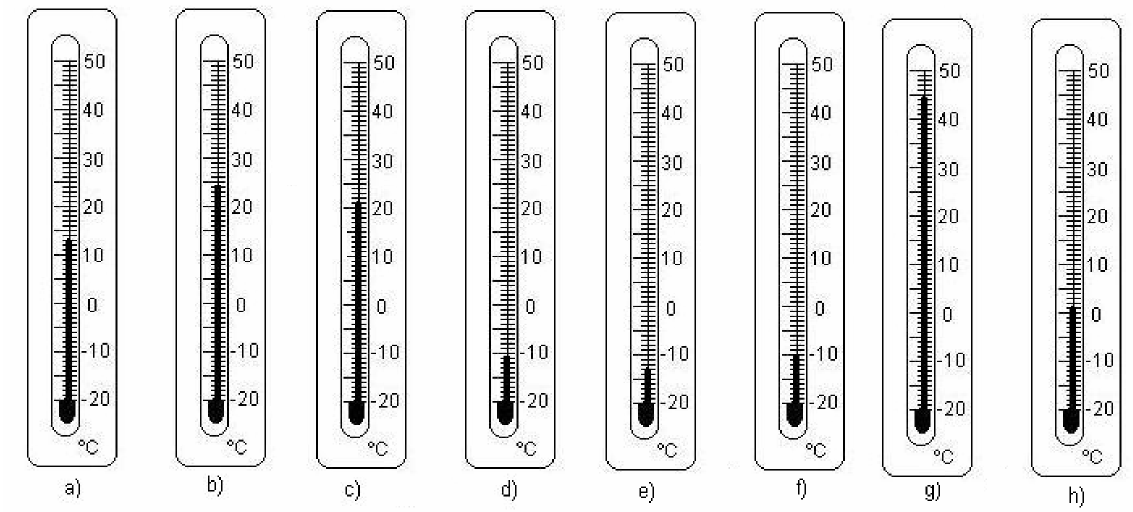
Examples:

9 > 4 Is read: nine is **‘greater than’** four -7 < 9 Is read: minus seven is **‘less than’** nine.

Exercise 1

a) Read: 6 > -9 -2 > -8 0 > -5

b) Write down the temperatures shown on these thermometers. Find their position in a number line.



Exercise 2 What is the temperature which is:

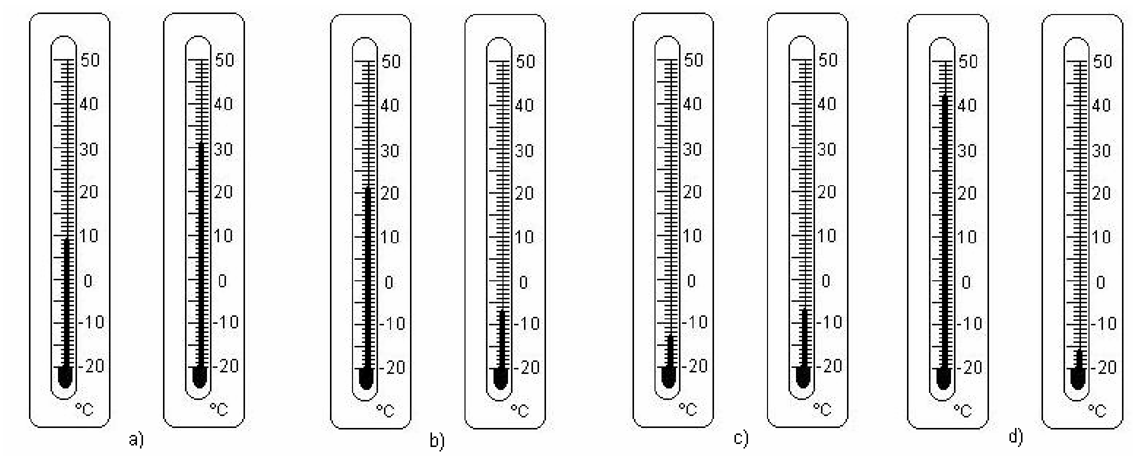
a) 7 degrees lower than 5º C

b) 6 degrees lower than 4º C

c) 16 degrees higher than - 4º C

d) 9 degrees lower than - 6º C

Exercise 3 What is the difference in temperature between each pair of thermometers?



Exercise 4 Write using integers.

a) Four degrees above zero

b) A withdrawal of 20.000 €

c) 250 meters below sea level

d) Three degrees below zero

e) A deposit of $200.00

f) An elevation of 8848 meters above sea level

g) A gain of 19kg

Exercise 5 Write the opposite of each integer given above.

2 Absolute Value of an Integer

The absolute value of any number is the distance between that number and zero on the number line.

.

If the number is positive, the absolute value is the same number.
If the number is negative, the absolute value is the opposite.

The absolute value of a number is always a positive number (or zero). We specify the absolute value of a number *n* by writing *n* in between two vertical bars: $|n|$.

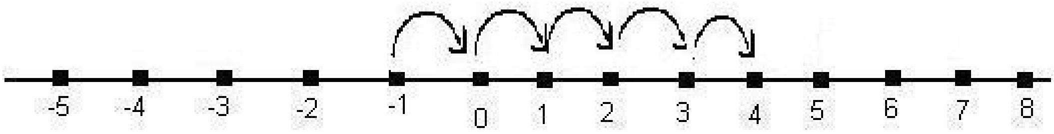
Examples:

$|6| = 6$ $|-10| = 10$ $|0| = 0$ $|123| = 123$ $|-3404| = 3404$

3 Adding Integers

There is a way to understand how to add integers. In order to add positive and negative integers, we will imagine that we are moving along a number line.

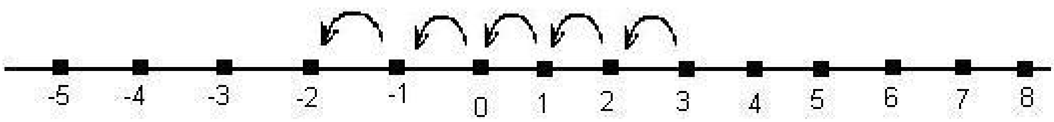
If we want to add -1 and 5, we start by finding the number -1 on the number line, exactly one unit to the left of zero. Then we would move five units to the right. Since we landed four units to the right of zero, the answer must be 4.



If asked to add 3 and -5, we can start by finding the number three on the number line (to the right of zero).

Then we move five units left from there because negative numbers make us move to the left side of the number line.

Since our last position is two units to the left of zero, the answer is -2.



Addition rules

When adding integers with the same sign

We add their absolute values, and give the result the same sign.

Examples:

2 + 5 = 7

(-7) + (-2) = -(7 + 2) = -9

(-80) + (-34) = -(80 + 34) = -114

With the opposite signs

We take their absolute values, subtract the smallest from the largest, and give the result the sign of the integer with the **larger** absolute value.

Example: 8 + (-3) = ?

The absolute values of 8 and -3 are 8 and 3.

Subtracting the smaller from the larger gives 8 - 3 = 5, and since the larger absolute value was 8, we give the result the same sign as 8, so 8 + (-3) = 5.

Example: 8 + (-17) = ?

The absolute values of 8 and -17 are 8 and 17.

Subtracting the smaller from the larger gives 17 - 8 = 9, and since the larger absolute value was 17, we give the result the same sign as –17

So 8 + (-17) = -9.

Example: -22 + 11 = ?

$$|-22| = 22$$
$$|11| = 11$$

Subtracting the smaller from the larger gives 22 - 11 = 11, and since the larger absolute value was 22, we give the result the same sign as -22

So -22 + 11 = -11.

Example: 32 + (-32) = ?

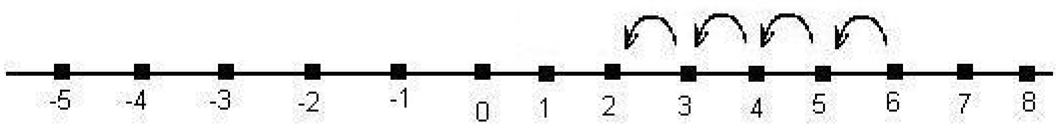
$$\left.\begin{array}{l} |-32| = 32 \\ |32| = 32 \end{array}\right\}$$

Subtracting the smaller from the larger, it gives 32 - 32 =0. The sign in this case does not matter, since 0 and -0 are the same. Note that 32 and -32 are opposite integers.

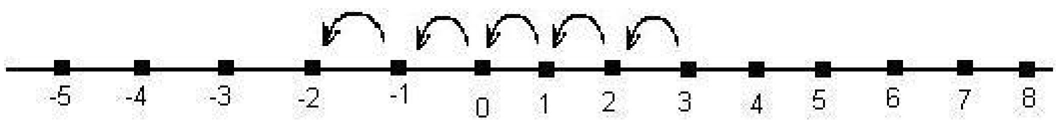
4 Subtracting Integers

To understand how to subtract integers, we will imagine that we are moving along a number line.

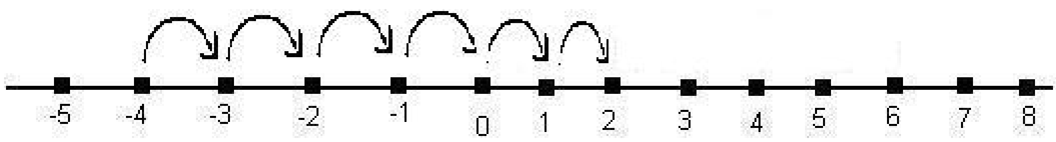
If we have to subtract 6 and 4, we start by finding the number six on the number line (6 units to the right of zero). Then we move four units to the left. Since we land two units to the right of zero, the answer is 2.



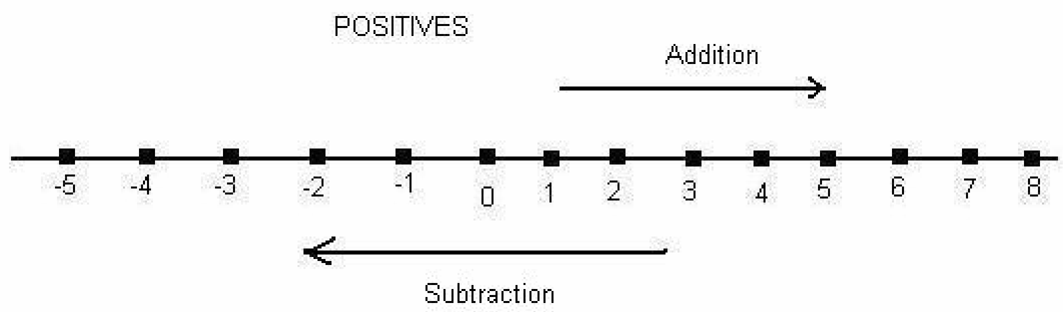
If we have to subtract 3 and 5, first find the number three on the number line, then move five units further left. Since we land two units left of zero, the answer is -2.

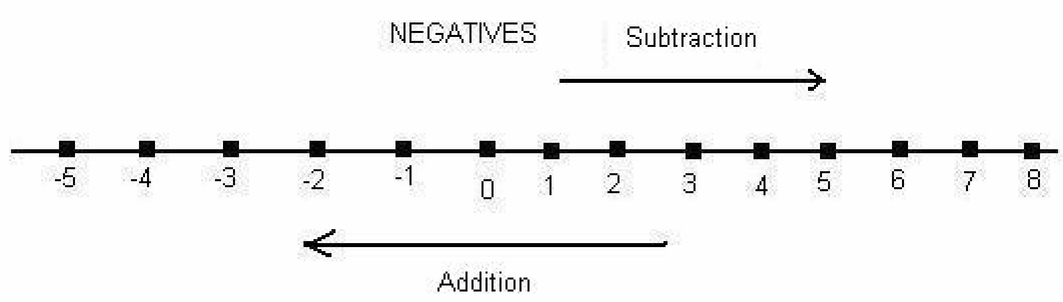


If we have to subtract -4 and -6, we start at -4, four units to the left of zero, and then we move six units to the right (**because we subtract a negative number!**). Since we land two units to the right of zero, the answer is 2.



So, to add and subtract positives or negatives numbers using the number line, you must move as is indicated below.





Subtraction rules

Subtracting an integer is the same as adding the opposite.

We convert the subtracted integer to its opposite, and add the two integers.

Examples:

7 - 4 = 7 + (-4) = 3 12 - (-5) = 12 + (5) = 17 -8 - 7 = -8 + (-7) = -15

-22 - (-40) = -22 + (40) = 18

The result of subtracting two integers could be positive or negative.

Exercises

Exercise 6 Write and solve an addition for each of the following sentences.

a) The temperature rises 6 degrees, and then falls 3 degrees.

b) You deposit $50.5 into the bank and then withdraw $38.25

c) The temperature falls 10 degrees, and then falls 4 degrees.

d) You fall 45.3 metres down a mountain, and then climb up 23.5 metres, but you fall again 15 metres.

Exercise 7 Add the following.

a) 5+(-3)=

b) (-12)+(-8)=

c) (-6)+12=

d) 15+(-19)=

e) (-5)+(-6)=

f) 22+(-43)=

g) (-5)+9=

h) 3+5=

i) (-4)+(0)=

Exercise 8

8.1 Add the following.

a) 18+(-20)+6=

b) (-13)+(-9)+(+5)=

c) (-7)+(-5)+(-4)=

d) (-17)+8+(-4)=

e) (8)+(-30)+22=

f) (-6)+5+(-6)=

g) (-9)+7+(-4)+6=

h) 7+(-13)+(-4)+(-5)=

8.2. Remove brackets that are not needed, rewrite the subtraction sentence and solve.

a) (+6)-(+8) =

b) (-4)-(-3) =

c) (+6)-(+9) =

d) (+3)-(-2) =

e) (+4)+(-3)+(-6)-(+7)+(+6) =

f) (-8)-(+5)+(+7)+(-8) =

8.3.Subtract.

a) 6-8 =

b) -7-3 =

c) 4-(-3)=

d) 8-(-8)=

e) -3-6 =

f) -12-8 =

g) -7-(-9)=

h) -6 - 4 =

8.4. Subtract.

a) 5-(-3)=

b) 5-4=

c) -8-4=

d) -3-6=

e) -9-(-5)=

f) 9-(-4)=

g) -5-3=

h) -2-(-4)=

8.5. Subtract.

a) –3 + (-4) –2 =

b) – 6 - (-8) – 9 =

c) –12 - (-4) + (-3) =

d) –7 + (-3) + (-2) =

5 Multiplying Integers

To multiply a pair of integers:

If both numbers have the same sign (positive or negative)

Their product is the product of their absolute values (their product is positive)

These examples help us to understand why a positive number is always the result of multiplying two numbers of the same sign.

Negative times negative: Make a pattern. Look at these sequences and complete them :

Example 1
-2 x 3 = -6
-2 x 2 = -4
-2 x 1 = -2
-2 x 0 = 0
-2 x -1 = 2
-2 x (-2) =
-2x(-3) =
-2x(-4) =

Example 2
-3 x 3 = -9
-3 x 2 = -6
-3 x 1 = -3
-3 x 0 = 0
-3 x (-1) =
-3 x (-2) =
-3 x(-3) =
-3 x(-4) =

Example 3
(-4) x 3 =
(-4) x 2 =
(-4) x 1 =
(-4) x 0 =
(-4) x (-1) =
(-4) x (-2) =
(-4) x (-3) =
(-4) x (-4) =

If the numbers have opposite signs

their product is the *opposite* of the product of their absolute values (their product is negative). If one or both of the integers is 0, the product is 0.

Look at the following chart below.

PRODUCT	+	-
+	POSITIVE	NEGATIVE
-	NEGATIVE	POSITIVE

Note: Multiplying integers of the **same sign** will give a **positive number**.

Multiplying integers with **opposite sign** will give a **negative number.**

Examples:
4× 3 = 12 Both numbers are positive, so we just take their product.

(-4) × (-5) = |-4| × |-5| = 4 × 5 = 20 Both numbers are negative, so we take the product of their absolute values.

In the product of (-7) × 6, the first number is negative and the second is positive, so we take the product of their absolute values, which is |-7| × |6| = 7 × 6 = 42, and give this result a negative sign: -42

So (-7) × 6 = -42

In the product of 12 × (-2), the first number is positive and the second is negative, so we take the product of their absolute values, which is
|12| × |-2| = 12 × 2 = 24, and give this result a negative sign: -24

So 12 × (-2) = -24.

To multiply any number of integers:

1. Count the number of negative numbers in the product.

2. Take the product of their absolute values.

If the number of negative integers counted in step 1 is **even**, the product is just the product from step 2 (**positive).**

If the number of negative integers is **odd,** the product is the opposite of the product in step 2 (give the product in step 2 a **negative** sign).

If any of the integers in the product is 0, the product is 0.

Example: 4 × (-2) × 3 × (-11) × (-5) = ?

Counting the number of negative integers in the product, we see that there are 3 negative integers: -2, -11, and -5.
Next, we take the product of the absolute values of each number:
4 × |-2| × 3 × |-11| × |-5| = 1320.
Since there were an odd number of integers, the product is the opposite of 1320, which is –1320

So 4 × (-2) × 3 × (-11) × (-5) = -1320.

Example: *-5* × (-2) × (-3) × (-1) × (4) = ?

Counting the number of negative integers in the product, we see that there are 4 negative integers
Next, we take the product of the absolute values of each number:
5 × 2 × 3 × 1 × 4 = 120.
Since there is an even number of integers, the product is 120.

Exercise 9 Multiply the following.

a) 5x(-13) =

b) (-12)x(-8) =

c) (-16)x12 =

d) 15x(-19) =

e) (-5)x(-6) =

f) 12x(-3) =

g) (-15)x6 =

h) 3x35 =

i) (-4)x(0)=

k) -3x(-5) =

Exercise 10

10.1 Multiply .

a) 7x(-12)x6=

b) (-13)x(-9)x(+5)=

c) (-3)x(-7)x(-14)=

d) (-6)x8x(-4)=

e) (9)x(-30)x2=

f) (-13)x5x(-6)=

g) (-9)x17x(-4)x26=

h) 4x(-81)x(-4)x(-2)=

10.2. Multiply

a) (+6) ·(+18) =

b) (-12) ·(-3) =

c) (+6) ·(+7) =

d) (+7) ·(-6) =

e) (+3) ·(-3) ·(-2) ·(+6) ·(+3) =

f) (-6) ·(+9) ·(+3) ·(-1) =

10.3. Operate

a) –3 ·[(-4) –2] =

b) – 6 ·[- (-8) – 9] =_

c) [–12 - (-4)] ·(-3) =

d) (–7 + (-3)) ·(-2) =

6 Dividing Integers

To divide a pair of integers the rules are the same as the rules for the product:

If both numbers have the same sign (positive or negative)

> Divide the absolute values of the first integer by the absolute value of the second integer (the result is positive)

If the numbers have opposite signs

> Divide the absolute value of the first integer by the absolute value of the second integer, and give this result a negative sign.

The chart is.

DIVISION	+	-
+	POSITIVE	NEGATIVE
-	NEGATIVE	POSITIVE

Note: Dividing integers of the **same sign** will give a **positive number**.

Dividing integers with **opposite sign** will give a **negative number.**

Examples:

$14 \div 2 = 7$ Both numbers are positive, so we just divide as usual.

$(-24) \div (-3) = |-24| \div |-3| = 24 \div 3 = 8$. Both numbers are negative, so we divide the absolute value of the first by the absolute value of the second.

$(-100) \div 25$ The numbers have different signs, so we divide the absolute value of the first number by the absolute value of the second.

$|-100| \div |25| = 100 \div 25 = 4$, and give this result a negative sign: -4

So $(-100) \div 25 = -4$

$98 \div (-7)$ Both numbers have different signs, so we divide the absolute value of the first number by the absolute value of the second.

$|98| \div |-7| = 98 \div 7 = 14$, and give this result a negative sign: -14.

So $98 \div (-7) = -14$.

Exercise 11 Divide the following.

a) 15:(-3)=

b) (-12):(-4)=

c) (-16):2=

d) 150:(-6)=

e) (-225):(-15)=

f) 12:(-3)=

g) (-156):6=

h) 35:(-35)=

i) (-4):(0)=

Exercise 12

12.1 Divide .

a) 12:(6:(-2))=

b) (-30):[(-15):(+5)]=

c) (-8):[(-14):(-7)]=

d) [(-6)x8]:(-4)=

e) [(9)x(-30)]:2=

12.2. Divide

a) (+76):(+18) =

b) (-12):(-3) =

c) (+42):(+7) =

d) (+72):(-6) =

12.3. Operate

a) –30: [(-4) –2] =

b) – 34 :[(-8) – 9] =

c) [–12 - (-3)]:(-3) =

d) (–7 + (-3)):(-2) =

Order of operations

- Do of all the operations in brackets first.

- Then do multiplication and division in the order they appear, then do addition and subtraction in the order they occur

Easy way to remember them

- Brackets
- Exponents
- Division
- Multiplication
- Addition
- Subtraction
- This gives you **BEDMAS.**

Do one operation at a time.

Example
[10+(-16)+(-24)]/3 = [-6+(-24)]/3 = -30/3= -10

13. Operate

a) (+3):(-3)+(-12):(+6)+ (+3) =

b) 62/[-10+(-4)-4] =

c) $(-3)\cdot(-4)-(-24):6+15:(-3)=$

d) $10:(-2)-(-7)\cdot(-3)+4=$

e) $1+\left[(-3)/\left((-5)+2\right)\right]-35/(-7)=$

f) $4-\left[7+(-3)\cdot(-2)\right]+(-72):(-9)=$

EXTRA EXERCISES

1 Mount Everest is 29,028 feet above sea level. The Dead Sea is 1,312 feet below sea level. What is the difference of altitude between these two points?

2 The temperature in Chicago was 4º C at two in the afternoon. If the temperature dropped 12º C at midnight, what is the temperature now?

3 A submarine was situated 2100 feet below sea level. If it ascends 1230 feet, what is its new position?

4 Aristotle was born in 384 B.C. and died in 322 BC. How old was he when he died?

5 A submarine was situated 1230 feet below sea level. If it descends 125 feet, what is its new position?

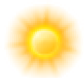
6 This is the three-day forecast for Yellowknife (Canada) from the 24th of November 2008

Today	Tue	Wed
Nov 24	Nov 25	Nov 26
		
Snow	Cloudy	Snow
-6°	-7°	-6°
-13°	-8°	-14°

What is the difference between the maximum and the minimum temperatures each day?

What are the maximum and the minimum temperatures during these three days?

7 This is the three-day forecast for Birmingham, UK from the 24th of November 2008

Today	Tue	Wed
Nov 24	**Nov 25**	**Nov 26**
		
Rain	Sunny	Partly Cloudy
13°	**14°**	**14°**
2°	**-2°**	**1°**

What is the difference between the maximum and the minimum temperatures each day?

8 The Punic Wars began in 264 B.C. and ended in 146 B.C. How long did the Punic Wars last?

9 This is a table with the melting points of some metals

Metal	Melting point º C	Boiling point º C
Aluminium	**660.32**	**2519**
Iron	**1538**	**2861**
Gold	**1064.18**	**2856**
Mercury	**-38.83**	**656.73**

a) **Calculate the difference between the melting and the boiling point of each metal**

b) **How much warmer is the melting point of mercury than the melting point of iron?**

10 On the 2nd of January, the temperature dropped from 3ºC at two o'clock in the afternoon to -11ºC at 8 a. m. the next day. How many degrees did the temperature fall?

11 A Greek treasure was buried in the year 164 BC and found in 1843 AD. How long was the treasure hidden?

12 On the 1st of December, the level of the water in a reservoir was 130 cm above its average level. On the 1st of July it was 110 cm below its average level. How many cm did the water level drop in this time?

7 Measurements

1 Measurement

Different units of measurement have been used in the past, some of them are still in use in UK and USA, but in most places is used the **Metric System of Measurements.**

The metric units are:

Length	metre (In U.S.A. “meter”)
Area	square metre
Volume	cubic metre
Capacity	litre
Mass or weight	gram

It is very useful to be able to *estimate* lengths, masses, etc. because it may not always be easy to measure them. Some useful hints for estimating are listed below:

The height of a standard door is about 2 m.
The length of an adult pace is about 1 m.
The length of a size 8 (41 in Spain) shoe is about 30 cm.
Most adults are between 1.5 m and 1.8 m in height.
It takes about 15 minutes to walk one kilometre.
The mass of a standard bag of sugar is 1 kg.
The mass of a family car is about 1 tonne.
2 hectares = 20 000 m^2 (about 3 football pitches).
A teaspoon holds about 5 ml of liquid.
The volume of a normal can of drink is about 330 cm^3.

In England some imperial units are in common use today. People may for example give their height in feet and inches; give a distance in miles, their weight in stones or a capacity in gallons. We will see the relationships between the units of the **Imperial** and the **Metric** systems of measurement.

2 Metric Prefix Table

The metric (decimal) system uses a number of standard prefixes for units of length, mass, etc.

To change any of the other units of measurement into their **equivalent values in the main unit we have to** use the conversion factor given by the value of the prefix.

Remember:

Number	Prefix	Symbol
10	deca	da
100	hecto	h
1000	kilo	k
1,000,000	mega	M
10^{9}	giga	G
10^{12}	tera	T

Number	Prefix	Symbol
0.1	deci	d
0.01	centi	c
0.001	mili	m
0.000001	micro	µ
0.000000001	nano	n
0.000000000001	pico	p

3. Length

The main unit of length is the metre; it is very useful to be able to convert between different units. This is the conversion table:

	Kilometre	Hectometre	Decametre	Metre	Decimetre	Centimetre	Millimetre
Kilometre **km**	1	10	100	1000	10000	100000	1000000
Hectometre **hm**	0.1	1	10	100	1000	10000	100000
Decametre **dam**	0.01	0.1	1	10	100	1000	10000
Metre **m**	0.001	0.01	0.1	1	10	100	1000
Decimetre **dm**	0.0001	0.001	0.01	0.1	1	10	100
Centimetre **cm**	0.00001	0.0001	0.001	0.01	0.1	1	10
Millimetre **mm**	0.000001	0.00001	0.0001	0.001	0.01	0.1	1

Exercise 1 Measure the width of this page and write it in the seven different units of the table.

Exercise 2 Write all the following in centimetres.

a) **4 cm 2 mm**

b) **18 cm 9 mm**

c) **75 mm**

d) **4 dm 3 cm 45 mm**

e) **7.8 m 43 dm**

f) **55.3 m**

g) **0.3 dam 5 cm 64 mm**

h) **0.05 hm 5 m 36 cm**

i) **4.6 km 0.3 dam 0.5 m 78 mm**

Exercise 3 Write all the following in millimetres.

a) **0.4 cm 12 mm**

b) **1.78 cm 15 mm**

c) **17.5 m**

d) 35 dm 13 cm 67 mm

e) 17.4 m 45 dm

f) 75.8 m

g) 0.7 dam 5 m 64 mm

h) 1.05 hm 15 m 36 cm

i) 0.06 km 0.3 dam 1.6 m 38 dm

Exercise 4 Write all the following in metres.

a) 10.4 cm 140 mm

b) 198 cm 150 mm

c) 17.5 km

d) 37 hm 13 m 1067 mm

e) 32.6 dam 470 dm

f) 1275.8 mm

g) 0.29 dam 5 m 765 mm

h) 1.32 hm 150 m 3600 cm

i) 0.005 km 0.12 dam 1.6 m 38 cm

Exercise 5 Write all the following in kilometres.

a) 10700 cm 140000 mm

b) 158 m 120000 mm

c) 17.5 hm

d) 346 hm 14m 10400 mm

e) 320.9 dam 47000 dm

f) 1275.8 dam

g) 8.78 dam 500 m 775000 mm

h) 43.32 hm 150000 m

i) 0.005 km 14 dam 160 m 38000 cm

Exercise 6 Round each of the following measurements to the nearest centimetre.

a) 3.84 cm

b) 158 m 1.2 mm

c) 415.5 mm

d) 134 m 19 mm

e) 342 dam 0.47 dm

f) 1273.8 mm

Exercise 7 Round each of the following measurements to the nearest metre.

a) 13.84 dm

b) 333 m 12 dm

c) 2473.5 mm

d) 543 m 900 mm

e) 78.9 dam 47.47 dm

f) 7654.8 mm

4. Capacity

The main unit of capacity is the litre; this is the conversion table:

	Kilolitre	Hectolitre	Decalitre	Litre	Decilitre	Centilitre	Millilitre
Kilolitre **kl**	1	10	100	1000	10000	100000	1000000
Hectolitre **hl**	0.1	1	10	100	1000	10000	100000
Decalitre **dal**	0.01	0.1	1	10	100	1000	10000
Litre **l**	0.001	0.01	0.1	1	10	100	1000
Decilitre **dl**	0.0001	0.001	0.01	0.1	1	10	100
Centilitre **cl**	0.00001	0.0001	0.001	0.01	0.1	1	10
Millilitre **ml**	0.000001	0.00001	0.0001	0.001	0.01	0.1	1

Exercise 8 Write all the following in centilitres.

a) 14 cl 32 ml

b) 4 dl 3 cl 45 ml

c) 0.38 dal 5 cl 68 ml

Write all the following in litres.

a) 90.5 hl 5 l 36 cl

b) 15.6 kl 0.03 dal 3.6 l 668 ml

c) 4.2 kl 0.53 dal 0.5 l 780 ml

Write all the following in kilolitres.

a) 43107cl 670140000 ml

b) 73.39 hl 17000 l

Exercise 9 Round each of the following measurements to the nearest litre.

a) 16.84 dl

b) 543 l 640 ml

c) 127 l 98 dl

d) 2283.5 ml

5. Weight

The unit of weight is the gram, this is the conversion table:

	Kilogram	Hectogram	Decagram	Gram	Decigram	Centigram	Milligram
Kilogram **kg**	1	10	100	1000	10000	100000	1000000
Hectogram **hg**	0.1	1	10	100	1000	10000	100000
Decagram **dag**	0.01	0.1	1	10	100	1000	10000
Gram **g**	0.001	0.01	0.1	1	10	100	1000
Decigram **dg**	0.0001	0.001	0.01	0.1	1	10	100
Centigram **cg**	0.00001	0.0001	0.001	0.01	0.1	1	10
Milligram **mg**	0.000001	0.00001	0.0001	0.001	0.01	0.1	1

1 tonne = 1000 kg

Exercise 10 Find your own weight in all the units.

Exercise 11 (You must be groups of four). Calculate the average weight of the group.

Round the result to the nearest

a) **kg**

b) **hg**

c) **dag**

Exercise 12 The average weight of a group of ten boys is 72 kilograms 890 grams. When one boy leaves the group the average of the nine becomes 72.5 kg.

Find the weight of the boy who left the group.

Exercise 13 A bridge has been designed to support 550 tonnes. If the average weight of a vehicle is 1 tonne 850 kg, is it safe to have 300 vehicles on the bridge at one time?

6. Area

The S I unit of area is the **square metre**. To change any of these other units of area into their **equivalent values in square metres** use the operation given.

square	Kilometre	Hectometre	Decametre	Metre	Decimetre	Centimetre	Millimetre
Kilometre **km^2**	X 1	X 10^2	X 10^4	X 10^6	X 10^8	X 10^{10}	X 10^{12}
Hectometre **hm^2**	: 10^2	1	X 10^2	X 10^4	X 10^6	X 10^8	X 10^{10}
Decametre **dam^2**	: 10^4	: 10^2	1	X 10^2	X 10^4	X 10^6	X 10^8
Metre **m^2**	: 10^6	: 10^4	: 10^2	1	X 10^2	X 10^4	X 10^6
Decimetre **dm^2**	: 10^8	: 10^6	: 10^4	: 10^2	1	X 10^2	X 10^4
Centimetre **cm^2**	: 10^{10}	: 10^8	: 10^6	: 10^4	: 10^2	1	X 10^2
Millimetre **mm^2**	: 10^{12}	: 10^{10}	: 10^8	: 10^6	: 10^4	: 10^2	1

Land measurements units: **Are = dam^2, Hectare = hm^2**

Exercise 14 Write all the following in m^2.

a) **3 km^2 5 hm^2 54 dm^2**

b) **7,890 ha 23 a**

c) **30,000 dm^2**

Write all the following in ares and in hectares.

d) **90.5 km^2 5 hm^2 36 dam^2**

e) **1.6 dam^2 0.03 km^2 34500 dm^2**

f) **5,000,000 cm^2**

7. Volume

The distinction between 'Volume' and 'Capacity' is artificial and kept here only for historic reasons. A **cubic metre** is **1000 litres**, a **cubic decimetre** is a **litre,** and a **cubic centimetre** is a **millilitre.** This is the conversion table:

cubic	Kilometre	Hectometre	Decametre	Metre	Decimetre	Centimetre	Millimetre
Kilometre **km^3**	X 1	X 10^3	X 10^6	X 10^9	X 10^{12}	X 10^{15}	X 10^{18}
Hectometre **hm^3**	: 10^3	1	X 10^3	X 10^6	X 10^9	X 10^{12}	X 10^{15}
Decametre **dam^3**	: 10^6	: 10^3	1	X 10^3	X 10^6	X 10^9	X 10^{12}
Metre **m^3**	: 10^9	: 10^6	: 10^3	1	X 10^3	X 10^6	X 10^9
Decimetre **dm^3**	: 10^{12}	: 10^9	: 10^6	: 10^3	1	X 10^3	X 10^6
Centimetre **cm^3**	: 10^{15}	: 10^{12}	: 10^9	: 10^6	: 10^3	1	X 10^3
Millimetre **mm^3**	: 10^{18}	: 10^{15}	: 10^{12}	: 10^9	: 10^6	: 10^3	1

Exercise 15 Write all the following in m^3.

a) 0.0003 km^3 0.05 hm^3 5400 dm^3

b) 7,320 dm^3 5000 cm^3

c) 210,000 dm^3

Write all the following in cm^3 and dm^3

d) 0.0123 m^3 40 dm^3 45800 mm^3

e) 0.0000045 dam^3 0.323 m^3 0.234 dm^3

f) 5,000,000 mm^3

Exercise 16 Write all the following in litres and in centilitres.

a) 0.0000125 hm^3 5.4 dm^3

b) 0.000043 hm^3 50000 cm^3

c) 210,000 millilitres

Write all the following in cm^3 and dm^3

d) 10.7 kl 40 hl 44,300 l

e) 6.7 kl 234 cl

f) 5,000,000 ml

8. Imperial Units of Length, Capacity and Mass

The imperial system was used, until very recently, for all weights and measures throughout the UK. There are many aspects of everyday life where the system is still in common usage. Miles instead of kilometres are used.

The units of length are:

1 mile = 1609.344 metres
1 yard = 0.9144 metres
1 foot = 0.3048 metres
1 inch = 0.0254 metres

The relationships are:

from \ to	**miles**	**yards**	**feet**	**inches**
mile	1	1760	5280	63360
yard	(1/1760)	1	3	36
foot	(1/5280)	(1/3)	1	12
inch	(1/63360)	(1/36)	(1/12)	1

The following list gives some help to you.

- The height of a tall adult is about 6 feet.
- The width of an adult thumb is about 1 inch.
- The length of a size 41, 8 in England, shoe is about 1 foot.
- An adult pace is about 1 yard.

You will find the following abbreviations used for imperial units:

1 yard = 1 yd	6 feet = 6ft = 6'
9 inches = 9 in = 9 "	8 ounces = 8 oz
7 pounds = 7 lb	

Do not to use m as an abbreviation for miles because m is a standard abbreviation for metres.

For areas, apart from the correspondent to the length units, it is also used the **acre.**

1 acre	43560 **feet^2**	6272640 **inches^2**	4046.8… **meters^2**	(1/640) **miles^2**	4840 **yards^2**

In U.S.A. the conversion table (not all exact) for units of volume and capacity are:

from \ to	**feet^3**	**gallons**	**inches^3**	**litres**	**pints**	**quarts**	**yards^3**
foot^3		7.48	1728	28.3	59.8	29.9	(1/27)
gallon	0.13		231	3.78	8	4	0.0049
inch^3	(1/1728)	(1/231)		0.016	(1/29)	(1/57.75)	(1/47)
pint	0.0167	(1/8)	28.87	0.47		(1/2)	0.0006
quart	0.033	(1/4)	57.75	1.057	2		0.00123
yard^3	27	0.0049	46656	764	1615.79	807.896	

Remember that all these relationships are not exact.

But a gallon in U.K. are **4.546** litres, so for units of volume and capacity we have:

Common for U.S.A. and U.K.

1 gallon = 4 quarts = 8 pints

Only for USA

1 gallon = 3.78 litres
1 pint = 0.47 litres

Only for UK

1 gallon = 4.546 litres
1 pint = 0.57 litres

Some of the units of mass are:

1 ton (UK) = 1016 kg

1 ton (USA) = 907 kg

1 stone = 6.35 kg

1 pound = 0.453 kg

(Be careful 1 tonne is 1000 kg)

The following list gives some help to you.

- The mass of a bag of sugar is just over 2 pounds.
- The weight of an adult is between 10 and 16 stones
- Chocolate is sold in Spain in pounds and ounces.

The conversion table (not all exact) for units of weight is:

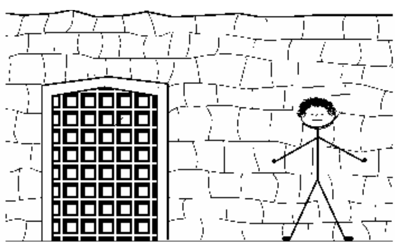
from \ to	**stone**	**pound**	**ounce**	**kg**
stone	1	14	224	6.35
pound	1/14	1	16	0.45
Ounce	1/224	1/14	1	0.028
Kg	0.16	2.2	35.3	1

Exercise 17

a) Estimate the length of the following line, in inches:

____

b) The picture shows a man standing with a gate in it: Estimate the height in feet of both the gate and the wall.



c) Estimate the size of the top of your desk, in inches.

d) Estimate the heights of 4 of your friends, in feet and inches.

e) Estimate the length and width of your classroom, in feet.

f) Estimate the total mass of 3 text books, in pounds.

g) Estimate the mass of an apple, in ounces. (Remember that there are 16 ounces in 1 lb.)

h) Estimate the capacity of a mug, in pints.

i) Estimate your own weight in stones and pounds.

Exercise 18 While on holiday in France, a family sees the following road-sign:

PARIS 342 km

How many *miles* is the family from Paris?

Exercise 19 While on holiday in England you see the following road-sign:

How many km/h is the speed limit?

Exercise 20 A bottle contains 2.5 litres of milk. How many *pints* of milk does the bottle contain?

Exercise 21 Vera buys 27 litres of petrol for her car. How many *gallons* of petrol does she buy? if she buys 4 gallons of petrol. How many litres does she buy?

Exercise 22 Change the following lengths into appropriate units of the metric system (cm, m or km).

4 feet

8 feet 7 inches

5 yards 2 feet

7 feet

5.5 feet

1 mile

4 feet 2 inches

2 yards

17 inches

95 inches

Change the following lengths into metres:

60 inches

29 inches

48 inches

240 inches

Exercise 23 Change the following masses into kg:

a) 7 pounds

b) 11 pounds

c) 36 pounds

d) 904 pounds

e) 2 stones

f) 9 stones 12 pounds

g) 5.5 pounds

Change the following masses into pounds or pounds and ounces:

a) 3.5kg

b) 500 g

c) 720 ounces

d) 750 g

e) 40 ounces

f) 125 g

Exercise 24 Change the following volumes into litres:

a) 5 gallons

b) 11 gallons

c) 63 gallons

d) 412 gallons

Exercise 25 Change the following volumes into gallons and litres:

a) 56 pints

b) 528 pints

c) 2 pints

d) 160 pints

e) 12 pints

Into gallons

f) 25 l

g) 120 l

h) 5.5 l

Exercise 26 A recipe requires 2 lb of flour. Give the equivalent amount of flour in:

a) Grams

b) Kilograms

c) Ounces

Exercise 27 The capacity of a fuel tank is 30 gallons. What is the capacity of the tank in:

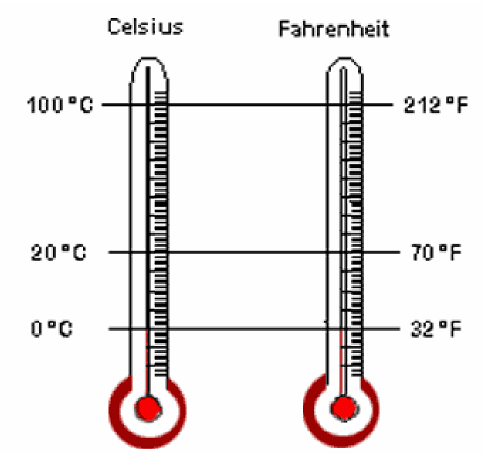
a) Litres

b) Pints

Exercise 28 A cow produces an average of 18 pints of milk each time she is milked. Convert this to litres, giving your answer to 1 decimal place.

9. Temperature

There are two main temperature scales, each one being named after the person who invented it.



G D FAHRENHEIT (1686-1736) a German physicist, in about 1714 proposed the first practical scale.
He called the freezing-point of water **32 degrees** (so as to avoid negative temperatures) and the boiling-point **212 degrees**.

Anders CELSIUS (1701-1744) a Swedish astronomer, proposed the 100-degree scale (from 0 to 100) in 1742.
This was widely adopted as the centigrade scale. But since grades and centigrade were also measures of angle, in 1947 it officially became the Celsius scale.

Nowadays, the CELSIUS scale is the preferred scale in our everyday lives.

However, the Fahrenheit scale is still widely used in U.S.A. and there frequently is a need to be able to change from one to the other.

To change temperature given in Fahrenheit (*F*) to Celsius (*C*)

1. Start with (***F***)
2. Subtract 32
3. Multiply by 5 and divide by 9
4. The answer is (***C***)

To change temperature given in Celsius (*C*) to Fahrenheit (*F*)

1. Start with (***C***)
2. Multiply by 9 and divide by 5
3. Add on 32
4. The answer is (***F***)

Example 1: **To convert 45º F to ºC**

1. 45º*F*
2. 45 – 32 = 13
3. (13x5)/9 = 7.2
4. 7.2º C

Example 2: **To convert 25º C to ºF**

1. 25º C
2. 25x9:5=45

3. 45+32 =77
4. 77º F

Exercise 29 Change the following temperatures into º C scale:

a) **72º F**

b) **100º F**

c) **93º F**

d) **5º F**

e) **– 52º F**

f) **Write the temperature of the human body in ºF**

g) **Write a list of 6 temperatures in ºC and ºF including the ones that you consider more interesting.**

Exercise 30 Change the following temperatures into º F scale:

a) **37º C**

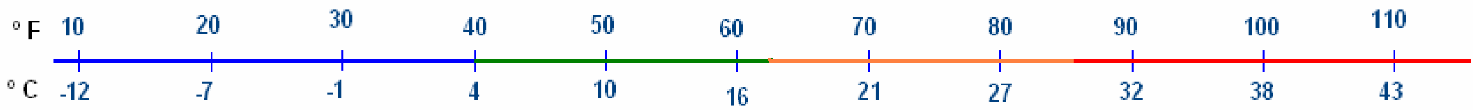
b) **100º C**

c) **18º C**

d) **– 5º C**

e) **– 22º C**

Compare your answers in this scale that is only an approximation.



8 Ratio and percentages

1 Ratio

A ratio is a comparison of two numbers. We generally separate the two numbers in the ratio with a colon (:) or as a fraction.

Suppose we want to write the ratio of 8 and 12.

We can write this as 8:12 or as 8/12, and we say the ratio is *eight to twelve*.

Examples:

Janet has a bag with 4 pens, 3 sweets, 7 books, and 2 sandwiches.

1. What is the ratio of books to pens?

Expressed as a fraction, the answer would be 7/4.
Two other ways of writing the ratio are 7 to 4, and 7:4.

2. What is the ratio of sweets to the total number of items in the bag?

There are 3 candies, and 4 + 3 + 7 + 2 = 16 items total.
The answer can be expressed as 3/16, 3 to 16, or 3:16.

2 Comparing Ratios

To compare ratios, write them as fractions. The ratios are equal if they are equal when written as fractions.

We can find ratios equivalent to other ratios by multiplying/ dividing both sides by the same number.

Example:

Are the ratios 2 to 7 and 4:14 equal?

The ratios are equal because 2/7 = 4/14.

The process of finding the simplest form of a ratio is the same as the process of finding the simplest form of a fraction.
1 : 3.5 or 2 : 7 could be given as more simple forms of the ratio 4 : 14.

Exercise 1 Simplify the following ratios:

3:6

25:50

40:100

9:21

11:121

For some purposes the best is to reduce the numbers to the form 1 : *n or n :* 1 by dividing both numbers by either the left hand side or the right-hand side number. It is useful to be able to find both forms, as any of them can be used as the unit in a problem.

Examples:

Which will be the divisor if we are to reach the form 1 : *n* for the ratio 4 : 5 ?
The divisor will be 4 and the ratio will be 1 : 1.25

And if we are to reach the form *n :* 1 for the same ratio?
The divisor will be 5, and the ratio will be 0.8 : 1

Exercise 2 Reduce to the form 1:n and n:1 10 shovels of cement, 25 shovels of sand

Exercise 3 Reduce to the form 1:n the following:

4:9

7:5

30:100

5:12

Exercise 4 Reduce to the form n:1 the following:

14:9

15:5

40:100

75:12

3. Direct Proportionality

We say that there is a direct proportionality between two magnitudes if when we double one magnitude, the other also doubles, when we half the first, the second also halves.

Examples:
If a can of cola costs 40p, the cost of:
2 cans is 80p
5 cans is £2.00

We can see that, for example, if we double the number of cans, we double the price. We say that the total cost of the cans increases proportionally with their number.

A proportion is one equality with a ratio on each side. It is a statement that two ratios are equal.

3/4 = 6/8 is an example of a proportion.

Note that proportions, ratios and equalities with fractions are different forms of expressing the same idea.

When one of the four numbers in a proportion is unknown, cross products may be used to find the unknown number. This is called solving the proportion. Letters are frequently used in place of the unknown number.

Example:

Solve for n: $1/2 = n/4$.
Using cross products we see that $2 \times n = 1 \times 4 = 4$, so $2 \times n = 4$. Dividing both sides by 2, $n = 4 \div 2$ so $n = 2$.

Exercise 5 Find the unknown side in each ratio or proportion:

1:7 = 5:x

1/9 = 5/x

1:5 = 3:x

2/7 = x/21

3:4 = 7:x

5/8 = 8/x

1/4=3/x

2 : 3 = 5 : x

12 : 21 = 16 : x

Exercise 6 Think of examples of proportionality in real life

Exercise 7

7.1 If 3 litres of petrol cost 3.45 €. How much will cost

a) **5 litres**

b) **23.5 litres**

7.2 If we travel 136 km in 1.5 hours driving at a constant speed.

a) How many km will we travel in 7.4 h?

b) How many hours do we need to travel 200 km?

7.3 Adrian finds that in each delivery of 500 bricks there are 20 broken bricks. How many bricks are broken in a delivery of 7500?

7.4 In a drink 53 ml of fruit juice are mixed with 250 ml of water. How many litres of water there are in 30l of that drink?

7.5 A car uses 25 litres of petrol to travel 176 miles. How far will the car travel using 44 litres of petrol?

4. Inverse Proportionality

Look at the relationship that exists between the number of the members of a family and the days that one box of apples lasts them (suppose that all the people eat the same amount of apples at the same rate).

Observe that the more people there are in the family the less time the box of fruit lasts, and the less people there are, the longer it lasts.

We will say that this relationship is of inverse proportionality

We say that there is an inverse proportionality between two magnitudes if when increasing one magnitude, (double, triple…) the other decreases (half, third…), when decreasing one (half, third…), the other increases (double, triple…).

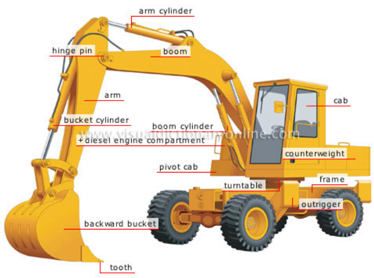
Exercises 8

8.1 A truck that carries 3 tons need 15 trips to carry a certain amount of sand. How many trips are needed to carry the same amount of sand with another truck that carries 5 tons?

8.2 An automobile factory produces 8100 vehicles in 60 days. With the production rhythm unchanged. How many units will be made in one year?

8.3 A driver takes $3\frac{1}{2}$ **hours to drive 329 km. How long will it take another trip in similar conditions as the previous one, but travelling 282 km instead?**

8.4 Two hydraulic shovels make the trench for a telephone cable in ten days. How long will it take to make the trench with 5 shovels?



5. Ratios with more than two parts

There are ratios with 3 or more parts.

Example: Tim, Tom and Tam are brothers. Tim has £10, Tom has £20 and Tam has £30. The money they have as a ratio is 10 : 20 : 30

We can simplify to 1 : 2 : 3

Exercise 9 Three angles are *a* = 60°, b = 80°, c = 80°. Write them as a ratio, and then simplify the ratio.

Can these be the angles of a triangle? If the answer is no, calculate the angles of a triangle with this ratio.

Exercise 10 The lengths of the sides of a quadrilateral are 4 cm, 4 cm, 6 cm and 6 cm. Find their ratio and simplify it.

T: What type of quadrilateral can it be if the sides are listed in order?

Exercise 11 Red blue and yellow paint are mixed in the ratio 3:2:7 to produce 6 litres of another colour. How much of each colour paint is used?

6. Percentages

A percent is a ratio of a number to 100. A percent is expressed using the symbol %.
A percent is also equivalent to a fraction with denominator 100.

Examples:

a) 5% of something = 5/100 of that thing.

b) 52% = 52/100 = 13/25 (nearly equals 1/2)

c) 8/200 is what %?

Method 1: 8/200 = (4 × 2)/(100 × 2), so 8/200 = 4/100 = 4%.

Method 2: Let 8/200 be n%. n = (8x100):200 = 4, so n% = 4%.

6.1 Percent as a decimal

Percent and hundredths are basically equivalent. This makes conversion between percent and decimals very easy.

To convert from a decimal to a percent, just move the decimal 2 places to the right.

Examples:

0.15 = 15 hundredths = 15%.

0.0006 = 0.06%

Converting from percent to decimal form is similar, only you move the decimal point 2 places to the left.

Examples:

Express 3% in decimal form.

Moving the decimal 2 to the left (and adding in 0's to the left of the 3 as place holders,) we get 0.03.

Express 97’5% in decimal form. We move the decimal 2 places to the left to get 0.975.

Exercise 12 Convert the following percentages to decimals.

a) 75% b) 1.74%

c) 3.7% d) 80%

e) 15% f) 0.6%

g) 9% h) 0.07%

Exercise 13 Convert the following decimals to percentages.

a) 0.5 b) 0.74

c) 0.35 d) 0.08

e) 0.1 f) 0.52

g) 0.8 h) 0.07

i) 0.04 j) 0.18

k) 0.4 l) 0.3

6.2 Common percentages

Some percentages are very common and it is useful to know them as fractions or decimals.

Some very simple percents are:

100% = 1 50% = 1/2 = 0.5 25% = 1/4 = 0.25

10% = 1/10 = 0.1 1% = 1/100 = 0.01

7 Calculations with percentages

7.1 Percentage of a quantity

To calculate the percentage of a quantity we must multiply it by the percent and divide by 100.

Example:

Calculate the 5% of 72

5% of 72 is $\frac{72 \cdot 5}{100} = 3.6$

Exercise 14 Calculate:

a) 15% of 540 **b) 3% of 320**

c) 5.3% of 7 **d) 6% of 5430**

7.2 Calculate the number when we know the Percentage

We must multiply the percentage by 100 and divide by the percent.

Example:

The 22% of a number is 66, which is the number?

The quantity is $\frac{66 \cdot 100}{22} = 300$

Exercise 15 Calculate the number if:

a) 95% of the number is 1020

b) 5% of the number is 7

c) 15.3% of the number is 500

d) 83% of the number is 1086

7.3 Express a ratio or a fraction as a percent

To write as a percent a ratio or a fraction , we may convert them into a number dividing and then multiply by 100

Example:

Calculate 27 out of 32 as a . We do $\frac{27}{32}\cdot 100 = 84.3\ \%$

Exercise 16 Express each of the following as percentages.

a) 8 out of 50

b) 3 out of 25

c) 8 out of 20

d) 3 out of 10

e) 6 out of 50

f) 6 out of 40

g) 12 out of 80

h) 84 out of 200

i) $\frac{23}{75}$

j) $\frac{3}{5}$

k) $\frac{102}{37}$

7.4 Estimating percentages

Estimating a percent is as easy as estimating a fraction as a decimal, and converting to a percent by multiplying by 100.

To estimate the percent of a number, we may convert the percent to a fraction, if useful, to estimate the percent.

Example:

Estimate 19 as a percent of 80.

As a fraction, $19/80 \cong 20/80 = 1/4 = 0.25 = 25\%$.

The exact percent is in fact 23.75%, so the estimate of 25% is only 1.25% off. (About 1 part out of 100.)

Exercise 17 Estimate, and later calculate the exact answer:

7 as a percent of 960.

12% of 72.

9.6% of 51.

7.6 Calculate a number increased with a percentage

We add to the number the percentage.

Example: Calculate the value of 320 increased by a 5%

1. The increase is 5% of 320 is $\frac{320 \cdot 5}{100} = 16$

2. The final value is 320 + 16 = 336

7.7 Calculate a number decreased with a percentage

We subtract to the number the percentage.

Example: Calculate the value of 320 decreased by a 12%

1. The decrease is 12% of 320 is $\frac{320 \cdot 12}{100} = 38.4$

2. The final value is 320 – 38.4 = 281.6

7.7 Percentage increase decrease

Percent increase and decrease of a value measure how that value changes, as a percentage of its original value.

Percentage *increases and decreases* are calculated using:

$$\text{Percentage increase} = \frac{\text{increase}}{\text{initial value}} \text{x}100$$

$$\text{Percentage decrease} = \frac{\text{decrease}}{\text{initial value}} \text{x} 100$$

Examples:

1. A collectors' comic book is worth 120 € in 2004, and in 2005 its value is 132 €. The increase in price is 12€; 12 is 10% of 120, so we say its value increased by 10%.

2. Amy is training for the 1500 meter run. When she started training she could run 1500 meters in 5 minutes and 50 seconds. After a year of practice her time decreased by 8%. How fast can she run the race now?

Her old time was $5 \times 60 + 50 = 350$ seconds, and 8% of 350 are 28, so she can run the race in $350 - 28 = 322$ seconds (5 minutes and 22 seconds).

3. A compact disc that sells for $12 is on sale at a 20% discount. How much does the disc cost on sale? The amount of the discount is 20% of $12, which is $2.40, so the sale price is $12.00 - $2.40 = $9.60.

Exercises 18

18.1 A magazine for teenagers sells 110000 copies each month. The company's president wants to increase the sales by 6%. How many extra magazines would they have to sell to reach this goal?

18.2 Chocolate bars normally cost 80 cents each, but are on sale for 40 cents each. What percent discount is this?

18.3 Movie tickets sell for $8.00 each, but if you buy 4 or more you get $1.00 off each ticket. What percent discount is this?

18.4 A firm decides to give 20% extra free in their packets of soap powder. How much extra soap powder would be given away free with packets which normally contain

(a) 2 kg of powder

(b) 1.2 kg of powder?

18.5 A house costs £30,000. A buyer is given a 10% discount. How much money does the buyer save?

18.6 Karen bought an antique vase for £120. Two years later its value increases by 25%. What is the new value of the vase?

18.7 When Wendy walks to school she covers a distance of 1800 m. One day she discovers a shortcut, which reduces this distance by 20%. How much shorter is the new route?

18.8 Kim's mother decides to increase her pocket money by 40%. How much extra does Kim receive each week if previously she was given £2.00 per week?

18.9 Express '30 out of 40' and '40 out of 50' as percentages. Which is the best score?

EXTRA EXERCISES

1. The price of a bar of chocolate is 35 c and includes 7 c profit. Express the profit as a percentage of the price.

2. In the crowd at a football match there were 35000 *Manchester United* supporters and 25000 *Manchester City* supporters. What percentage of the crowd supported each team?

3. A school won a prize of £1800. The Headmaster spent £750 on a new computer and the rest on software. What percentage of the money was spent on software?

4. James needs another 40 football stickers to complete his collection. There are a total of 500 stickers in the collection. What percentage of the collection does he have?

5. A 750 ml bottle of shampoo contains 200 ml of free shampoo. What percentage is free?

6. Adrian finds that in a delivery of 1500 bricks there are 50 broken. What is the percentage of broken bricks?

7. A glass of drink contains 50 ml of fruit juice and 200 ml of lemonade. What percentage of the drink is fruit juice?

8. Find each of the following, giving your answers to the nearest penny.

a) 30% of £150	**b) 12% of £903**	**c) 12.6% of £140**
d) 4.5% of £320	**e) 5.9% of £50**	**f) 8.2% of £18.**

9. A MP4 player has a normal price of £150.

a) In a sale its normal price is reduced by 12%. Find the sale price.
b) After the sale, normal prices are increased by 12%. Find the new price of the MP4 player.

10. Peter earns £9000 per year. He does not pay taxes on the first £3500 he earns and pays 25% taxes on the rest. How much tax does he pay?

11. A new gas supplier offers a 25% discount on the normal price and a further 5% discount if customers pay on line. For one client the gas bill is £130. Find out how much they have to pay after both discounts.

Remember

$$\text{Percentage increase} = \frac{\text{increase}}{\text{initial value}} \text{x}100$$

$$\text{Percentage decrease} = \frac{\text{decrease}}{\text{initial value}} \text{x}100$$

12. A baby weighed 5.6 kg and six weeks later her weight increases to 6.8kg. Find the percentage increase.

13. A factory produces blank DVDs at a cost of 88p and sells them for £1.10. Find the percentage profit.

14. A new car cost £11500 and one year later it is sold for £9995. Find the percentage reduction in the value of the car.

15. An investor bought some shares at a price of £14.88 each. The price of the shares drops to £11.45. Find the percentage loss.

16. A supermarket offers a £8 discount to all customers spending more than £40. Karen spends £42.63 and John spends £78.82. Find the percentage saving for each one.

17. In a year, the value of a house increases from £146 000 to £148 000. Find the percentage increase in the value of the house.

18. A battery powers an iPod for 12 hours. An improved version of the battery powers the iPod for an extra 30 minutes. Find the percentage increase in the life of the batteries.

9 Algebra and equations

1 Variables

A variable is a symbol that represents a number. We usually use letters such as x, n, p, t for variables.

Letters are useful if we want to operate with an unknown number instead with a particular one. Let us look at some examples:

We say that *s* represents the side of a square, then *s* represents a number and:

$4s$ Is the perimeter of the square

$s \cdot s$ Is the area of the square

When letters express numbers they have the same operating properties. The part of mathematics that deals with the study of the expressions with letters and numbers is called **algebra**.

2 Expressions

An expression is a mathematical statement with numbers and variables.

Examples:

3 x $x+3$ $2 \cdot (x-5)$ $x^2 - 3x$

If Mark weights 80 kg and he gains n kg, the new weight is 80 + n

Exercise 1 Calling '*a*' the age of a person write an expression for:

1.1 The age he/she will be in 2012.

1.2 The age he/she was 7 years ago

1.3 The age he/she will be after living the same time again.

Exercise 2 Calling x a number, express in algebra:

2.1 The sum of the number and 10

2.2 The difference between 123 and the number

2.3 The double of the number

2.4 The triple of the number plus three units

2.5 The half of the number minus seven

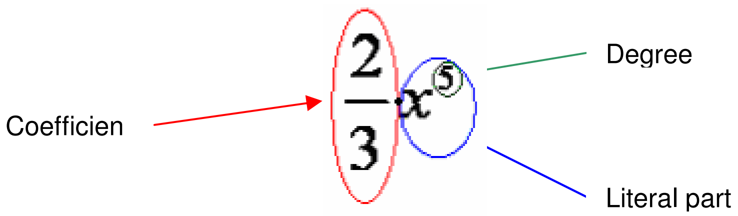
2.6 The three quarters of the number plus forty-six

3 Monomials

> The simplest algebraic expressions formed by products of numbers and letters are called monomial.

A monomial consists of the product of a known number (coefficient) by one or several letters with exponents that must be constant and positive whole numbers (literal part).

Generally in the monomials the product signs are not included, so we find a number followed by one or more letters, we understand that they are multiplied.



Examples:

a) 2x is a monomial. 2 is the coefficient, x is literal part.

b) $-3x^2$ is a monomial, -3 is the coefficient, x^2 is the literal part, x is the variable and the degree is 2

c) $\frac{3}{5}t^7$ is a monomial, $\frac{3}{5}$ is the coefficient, t^7 is the literal part, t is the variable and the degree is 7

d) $5xy^2$ is a monomial, 5 is the coefficient, xy^2 is the literal part, x and y are the variables and the degree is 3

e) $2x+7$ is an algebraic expression but it is not a monomial.

f) $\frac{3}{x}$ is an algebraic expression but it is not a monomial.

Exercise 3 Find which of the following expressions are monomials and determine, if they are so, their coefficient, literal part and variables:

a) $-\frac{1}{5}x^7$

b) $2t^2$

c) $a+b$

d) a^9

e) $\sqrt{2}n^4$

f) $3\sqrt{n}$

g) $7abc^2$

4 Operations of monomials

4.1 Addition or subtraction of two monomials.

We add or subtract the coefficients and we leave the literal part unchanged.

Two monomials can only be simplified when both have the same literal part, that is, when they are **"like monomials"**. When the literal part is different, the addition is left indicated.

Examples:

$5x + 7x = 12x$

$\frac{1}{2}x + 2x = \left(\frac{1}{2} + 2\right)x = \frac{5}{2}x$

$7ab + 2ab = 9ab$

Exercise 4 Operate

a) $2x - 7x =$ b) $15x + 2x =$ c) $8x - x =$

d) $\frac{15}{2}x + 2x =$ e) $-2x^2 + 23x^2 =$ f) $15x + 2 =$

g) $3a - 7a =$ h) $x^2 + 3x$ i) $x + \frac{2}{3}x =$

4.2 Product of two monomials.

We multiply the coefficients and also the literal part (remember how we multiply powers with the same base).

Examples:

$5x \cdot 7x = 35x^2$

$\frac{1}{2}x^2 \cdot 2x = x^3$

$7a^3 \cdot 2a^2 = 14a^5$

Exercise 5 Operate

a) $2x \cdot 7x =$ b) $15x \cdot 2x =$ c) $8x \cdot x =$

d) $\frac{15}{2}x \cdot 2x^3 =$ e) $-2x^2 \cdot 23x^2 =$ f) $15x \cdot 2 =$

5 Manipulating algebraic expressions

5.1 Evaluate an expression.

To evaluate an expression at some number means we replace a variable in the expression with the number and then, if necessary, we calculate the value.

Example:

Evaluate $5x+3$ when $x=2$, $5x+3$ becomes $5\cdot 2+3=13$

Remember the rules: **parenthesis, exponents, multiplications and divisions and, the last, additions and subtractions.**

Exercise 6 Evaluate the expressions:

a) $-\frac{1}{5}x^2$ when $x=3$

b) $2t^2$ when $t=2$

c) a+b when $a=b=7$

d) a^9 when $a=-1$

e) n^4+3n when $n=3$

f) $\frac{3n+2}{n-3}$ when $n=5$

g) $7a + b - c^2$ when $a = 3, b = -2, c = 4$

5.2 Expansion of brackets.

Multiplying a number by an addition is equal to multiplying by each adding number and then to adding the partial products.

Examples:

$2(x+6) = 2x + 12$

$3(2x-7) = 6x-21$

$3x(3x+2) = 9x^2+6x$

Exercise 7 Fill in the missing terms:

a) $5(3x-4) = \square - 20$ b) $x(x-2) = \square -2x$

c) $a(4-a) = \square - a^2$ d) $7x(3x-y) = \square - 7xy$

Exercise 8 Expand:

a) $2(x-4) =$ b) $6(2-x) =$

c) $6x(2x-1) =$ d) $2t(2t+8) =$

e) $5x(2x-3y) =$ f) $x(x-1) =$

g) $2(7+7x) =$ h) $2x(3x-a) =$

Exercise 9 Write down expressions for the area of these rectangles and then expand the brackets as in the example.

a) 3x+1

3 $A = 3(3x+1) = 9x + 3$

b) 2x-1

7 A =

c) 3x

x-2 A =

d) x-7

4 A =

5.3 Common factors

It is important to know how to write expressions including brackets when it is possible; this is called **factorization** or **factoring**

Examples:

$$12x + 6 = 6 \cdot 2x + 6 \cdot 1 = 6(2x + 1)$$
$$= 6(2x + 1)$$

$$21x - 14 = 7 \cdot 3x - 7 \cdot 2 = 7(3x - 2)$$
$$= 7(3x - 2)$$

$5x^2 - 8x = x(5x - 8)$ We can check our answer by expanding the expression.

Exercise 10 Factorise:

a) $10x + 15 =$ b) $4x - 12 =$ c) $9x - 9 =$

d) $3x^2 - 2x =$ e) $9x^2 + 3x =$ f) $3x - 3 =$

g) $33x^2 - 3x =$ h) $5x - 3x^2 =$ g) $13x^2 - 2 =$

6 Equations

An equation is a statement in which two expressions are equal.

- The letter in an equation is called the **unknown**. (Sometimes it is called the **variable**).

Examples of equations:

$x = 2$ $3 = x$ $2x = 4$ $5x+1 = 3$ $x^2 = 4$

In an equation we consider two members:

First member is the left member
Second member is the right member.

Each monomial is called a term. Monomials with the same literal part are called **“like terms”**

Example:

In the equation $3t - 2 = -7$ we say:

- The first member is $3t - 2$
- The second member is -7
- There are three terms 3t, -2 and –7
- The unknown is t.

In the equation $3x^2 - 6 = 5x + 2$:

- The first member is $3x^2 - 6$
- The second member is $5x + 2$
- There are four terms $3x^2$, -6, 5x and 2
- The unknown is x.

7 Solving equations

Solving an equation is to find out a solution.
A solution to an equation is the number that makes the equality true when we replace the unknown or variable with that number.

Examples:

2 is a solution to the equation $2x = 4$ because if we replace x by 2 that gives us $2 \cdot 2 = 4$, which is true.

3 is a solution to the equation $5x + 8 = 16$, if we replace x by 3, we have $5 \cdot 3 + 8 = 16$, which is true.

4 is not a solution to $3x - 1 = 4$ because $3 \cdot 4 - 1 = 4$ is not true.

7.1 Rules of simplifying and solving equations

These are the basic rules for simplifying equations:

These rules must be followed, usually in this order, to solve linear equations.

Rule 1 If there are brackets we must remove them (**expand the expression**) as in the usual operations with numbers.

Rule 2 We can add or subtract any number to or from both members of the equation so **any x-term or number that is adding (positive sign) moves to the other side subtracting (negative sign) and vice versa.**

Rule 3 Move x-terms to one side and numbers to the other.

Rule 4 Combine like terms

Rule 5 Any number **that is multiplying,** the whole expression, on one side moves to the other side **dividing,** and a number that is **dividing,** the whole expression, on one side moves to the other **multiplying**.

Examples:

1. Solve $5x + 12 - 4 = -2$

We add the two numbers in the first member, we get

$$5x + 8 = -2$$

We may subtract 8 from each member and make the addition of the integers numbers that appear

$$5x + 8 - 8 = -2 - 8,$$

$$5x + 0 = -2 - 8,\ 5x = -2 - 8$$

More easily , 8 that is positive moves to the right member as -8

$5x = -10$

We divide each member by 5 or we can say that 5 which is multiplying moves to the second member dividing $x = \frac{-10}{5}$

$x = -2$ we can check the solution (always in the original equation)

$5 \cdot 2 + 12 - 4 = -2$

$10 + 12 - 4 = -2$

$-2 = -2$ So our solution is correct.

2. Solve $7x + 5 = 2x$

We may subtract 5 from each member and make the addition of the integers numbers that appear

$7x + 5 - 5 = 2x - 5$

$7x = 2x - 5$

We may subtract 2x from each member

$7x - 2x = 2x - 2x - 5$

We can add like terms

We divide each member by 5

$5x = -5$

We divide each member by 5 and we get

$x = -1$ We check the solution

$7 \cdot (-1) + 5 = 2(-1)$

$-7 + 5 = -2$

$-2 = -2$ So our solution is correct.

3. Solve $2(x + 5) + 1 = 17$

We can expand

$2x + 10 + 1 = 17$

$2x + 11 = 17$

We may subtract 11 from each member

$2x + 11 - 11 = 17 - 11$ or simply 11 moves to the right member as -11 and then $2x = 17 - 11$

$2x = 6$

We divide each member by 2 or 2 which is multiplying moves to the right member dividing $x = 3$

We check the solution

$2(3 + 5) + 1 = 17$

$2 \cdot 8 + 1 = 17$

$17 = 17$ So our solution is correct.

Exercise 11 Solve:

1. $x + 7 = 10$
2. $x - 3 = 21$
3. $t - 6 = 7$
4. $x - 22 = 13 - 4$
5. $4 + y = 54$
6. $12 = 3 + z$
7. $m - 24 = 17$
8. $1 + p = 2 - 89$
9. $x + x = 37 - 2$
10. $23 + y + 2y = 3$
11. $4x = 29 - 1$

12. $100 = 16x + 20$

13. $5x + 3 = 20$

14. $10x = 13 - 2$

15. $2x + 7 = 6$

16. $2 = 7x - 3$

17. $12x - 7 = 13$

18. $3x - 5x = 23$
19. $5x + 7x = 23 - 1$

20. $44 + x = 12 - 3x$

Exercise 12 Solve:

1. $3(x - 2) = 7$

2. $2(x + 3) = 2$

3. $6(x - 5) = -2$

4. $7(x-1)+2x=20$

5. $3x-2(2x+3)=0$

6. $4(3x-5)-x=100$

7. $-21=6(3x-1)-13x$

8. $9(1-2x)+22x=1$

9. $2(3-x)+5x=1-3x$

10. $(3-x)\cdot 2-15x+2=3(1-3x)+2x-1$

Exercise 13 Express each problem as an equation and solve them

1. The sum of my age and 7 is 42. Find my age.

2. The difference of a number and 23 is 124. Find the number.

3. The quotient of 35 and a number is 7. Find the number.

4. If someone gives me 24€ I will have 34.23 € Find the money I have.

5. The double of my age minus 7 years is the age of my elder brother who is 19 years old.

6. The area of a square is 144 square metres. Find its length.

7. A teacher gives x coloured pencils each one of 8 girls except to one of them who only receives 5 pencils. The teacher gives 53 pencils in total. How many pencils did each girl receive?

8. Donald thinks of a number, multiplies it by 3 and subtracts 7. His answer is the double of the number plus 5 units. Which is the number?

9. In a triangle, the smallest angle is 20º less than the middle angle and the largest angle is twice the middle one. Find all the angles.

10. If we shorten 2 cm each of the two opposite sides of a square, we get a rectangle with an area which has 28 cm^2 less than the area of the square. Find out the perimeter of the rectangle.

10 Geometry, angles

1 Basic terms

Points

A point is one of the basic terms in geometry. We say that a point is a "dot" on a piece of paper. We identify this point with a number or letter. A point has no length or width.

Lines

A line is a "straight" line that we draw with a ruler on a piece of paper; a line extends forever in both directions.

Rays

A ray is a "straight" line that begins at a certain point and extends forever in one direction. The point where the ray begins is known as its endpoint.

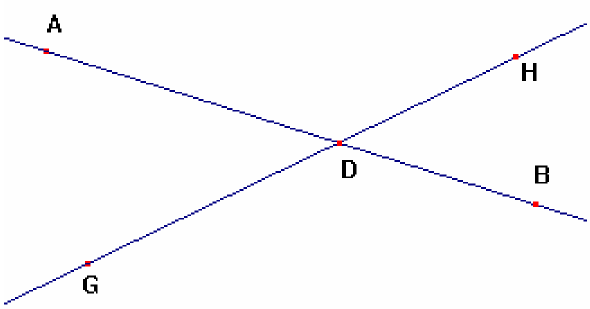
Line Segments

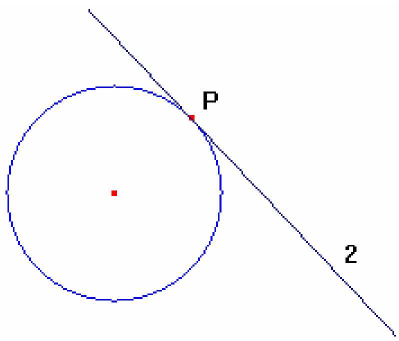
A line segment is a portion of a "straight" line. A line segment does not extend forever, but has two distinct endpoints. We write the name of a line segment with endpoints A and B as $\overline{AB}$.

Intersection

The term intersect is used when lines, ray lines or segments share a common point. The point they share is called the point of intersection.

Example: In the diagram below, line AB and line GH intersect at point D; line 2 intersects the circle at point P:

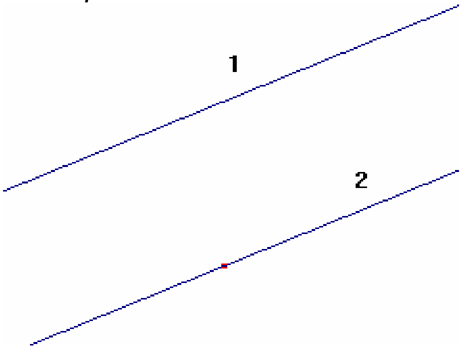




Parallel Lines

Two lines in the same plane which never intersect are called parallel lines. We say that two line segments are parallel if the lines that they lie on are parallel.

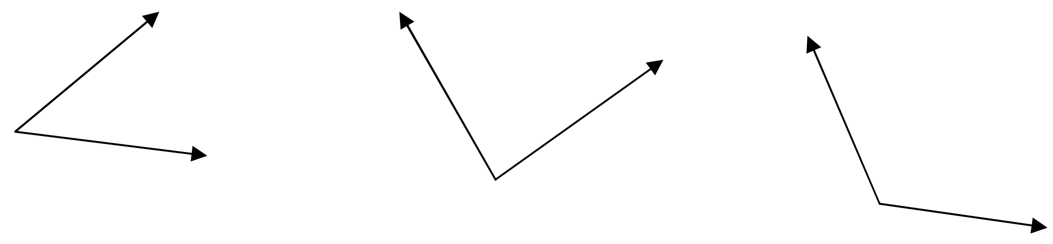
Examples: Lines 1 and 2 below are parallel.



The *opposite* sides AB, DC or AD, BC of the rectangle below are parallel.

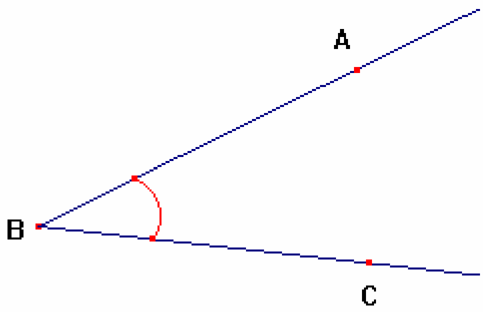
2 Angles

Two rays with the same endpoint form an angle. The point where the rays intersect is called the vertex of the angle. The two rays are called the sides of the angle.
Some examples of angles are:



2.1 How to name the angles.

We can name the angle below as $\angle B$ or even **b,** but it is better to name it as $\angle$ABC or as $\angle$CBA. Note how the vertex point is always given in the middle.

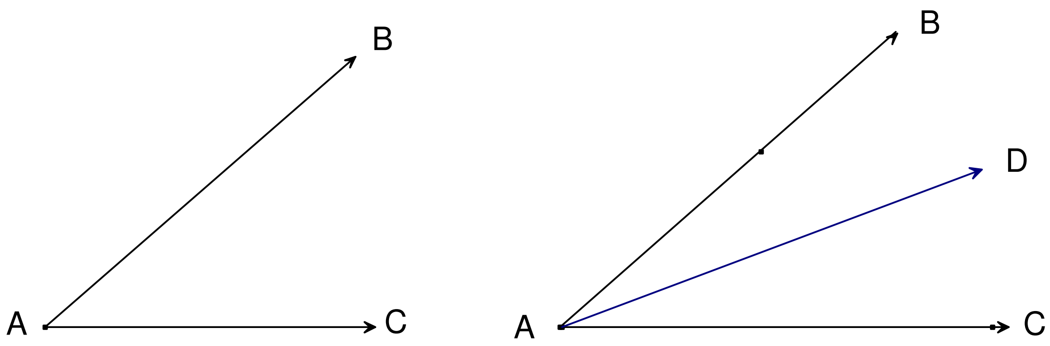


2.2 Angle Bisector

An angle bisector is a ray that divides an angle into two equal angles.

Example

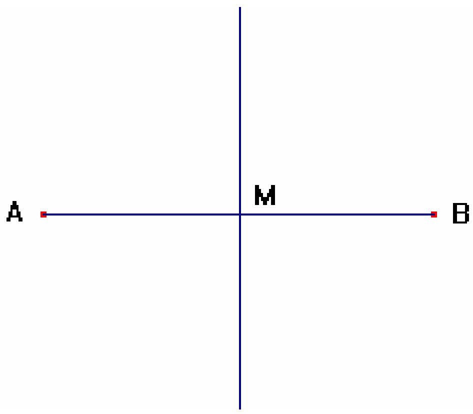
The ray AD on the right is the angle bisector of the angle BAC on the left because $\angle BAC = 2\cdot\angle BAD = 2\cdot\angle DAC$.



2.3 Bisecting line of a segment

Two lines that meet at a right angle are perpendicular.

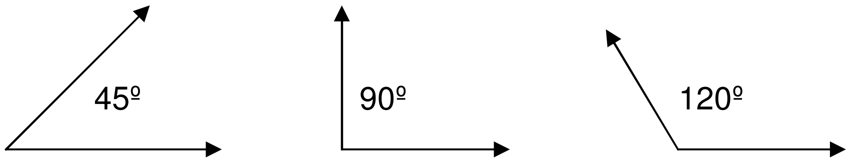
The perpendicular line that divides a segment into two equal parts is the bisecting line of the segment



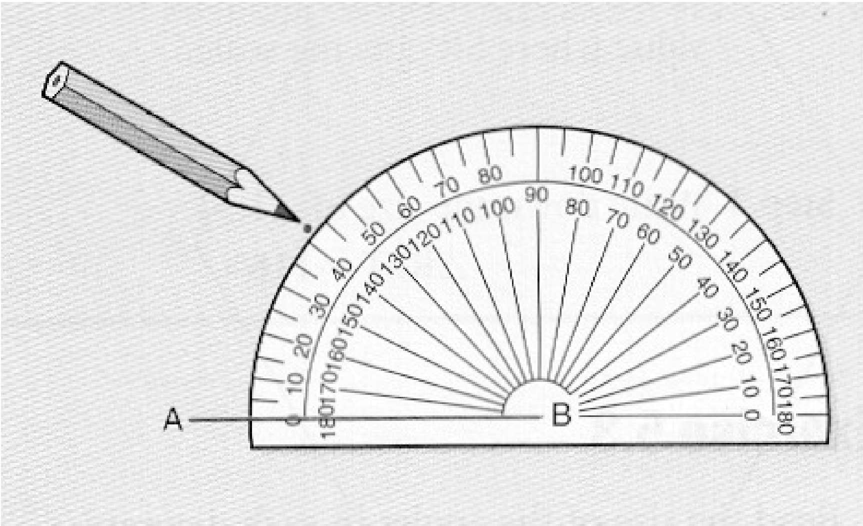
2.4 Measuring angles. Degrees

We measure the size of an angle using degrees.
There are 360º in a full turn.

Here are some examples of angles and their approximate degree measurements.



For drawing and measuring angles we use a **protractor**.



Note that usually there are two scales in the protractors, use the correct one.

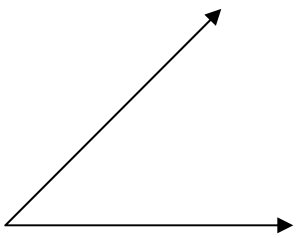
Exercise 1

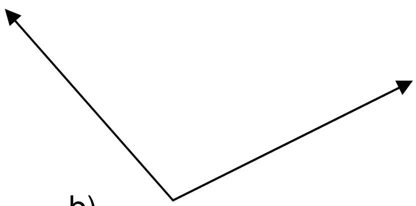
1.1 Draw in your notebook the following angles:

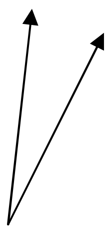
a) $\angle$ABC = 23º b) $\angle$CDE = 56º c) $\angle$PQR = 88º

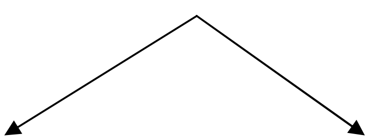
d) $\angle$ RST = 68º e) $\angle$KRT = 144º f) $\angle$MVL = 180º

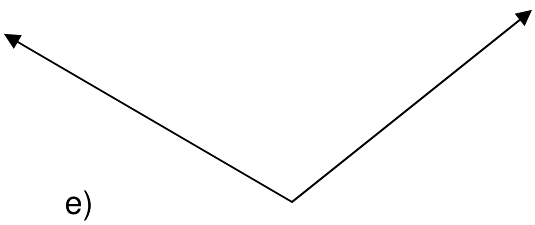
1.2 Measure the size of the following angles:

a) 

b) 

c) 

d) 

e) 

Approximations

- 1° is approximately the width of a pinky finger nail at arm's length
- 10° is approximately the width of a closed fist at arm's length.

3 Type of angles

Acute angles: An acute angle is an angle measuring between 0 and 90 degrees.

Obtuse angles: An obtuse angle is an angle measuring between 90 and 180 degrees.

Right angles: A right angle is an angle measuring 90 degrees. Two lines or line segments that meet at a right angle are perpendicular.

Straight angles: A straight angle is an angle measuring 180º

Complete turn: One complete turn is 360º

Exercise 2

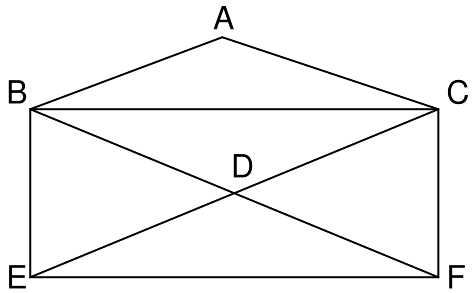
a) Make a list of objects in the classroom or in your house which contain:

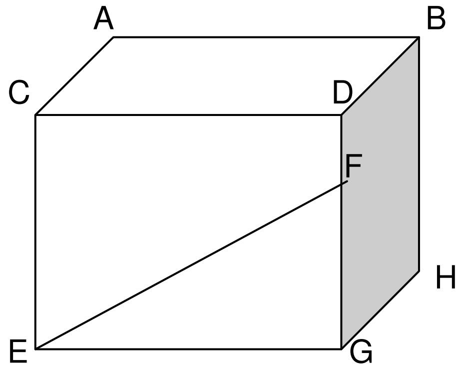
A right angle

A straight angle

An acute angle

b) Name all the obtuse, acute and right angles in the diagrams.





Exercise 3

How many degrees will the minute hand of a clock move in

a) 15 minutes **b) 30 minutes**

c) 5 minutes **d) 20 minutes**

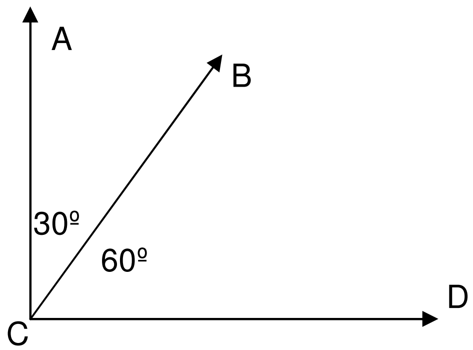
e) 1 minute **f) 46 minutes**

4 Related angles

Complementary Angles

Two angles are called complementary angles if the sum of their degree measurements equals 90 degrees. One of the complementary angles is the **complement** of the other.

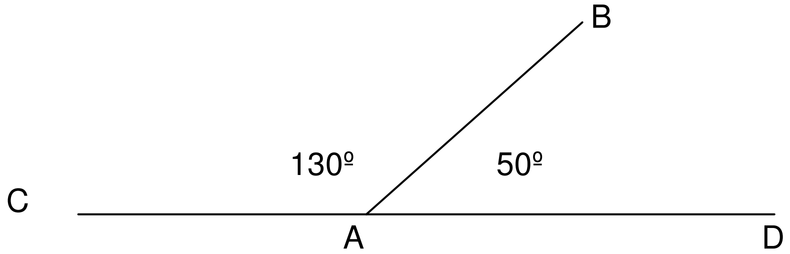
The two angles $\angle DCB = 60º$ and $\angle ACB = 30º$ are complementary.



Supplementary Angles

Two angles are called supplementary angles if the sum of their degree measurements equals 180 degrees. One of the supplementary angles is the **supplement** of the other.

The two angles $\angle DAB = 50º$ and $\angle CAB = 130º$ are supplementary.



Exercise 4 Calculate the complement and draw them on your notebook :

a) 17º

b) 42º

c) 7º

d) 90º

Exercise 5 Calculate the supplement of:

a) 37º

b) 52º

c) 123º

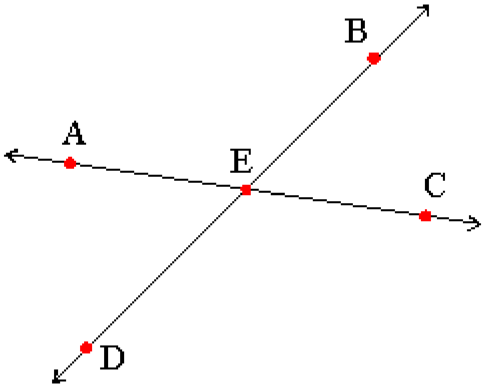
d) 7º

e) 90º

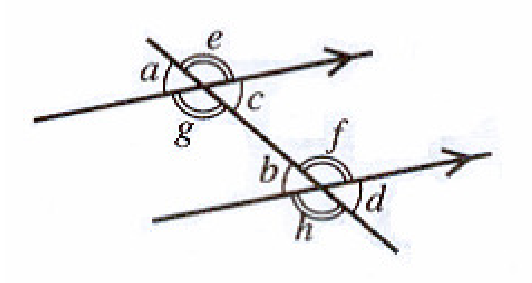
5 Angles between intersecting lines

Vertical Angles

For any two lines that meet, such as in the diagram below, angle $\angle BEC$ and angle $\angle AED$ are called vertical angles or vertical opposite angles. Vertical angles have the same degree measurement. Angle $\angle AEB$ and angle $\angle DEC$ are also vertical angles.



When a line crosses two parallel lines like in the diagram below



a = b = c = d

And

e = f = g = h

c and b are called **Alternate Interior Angles**

g and f are also **Alternate Interior Angles**

e and h are called **Alternate Exterior Angles**

a and d are also **Alternate Exterior Angles**

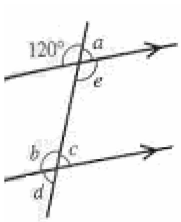
e and f are called **Corresponding Angles**

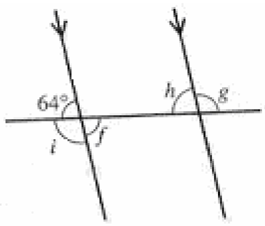
a and b are also **Corresponding Angles**

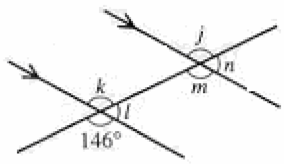
Remember that f and h or b and d, for example, are **Vertical Angles**

Exercise 6

a) Find the unknown angles and say all the correspondences you can see:





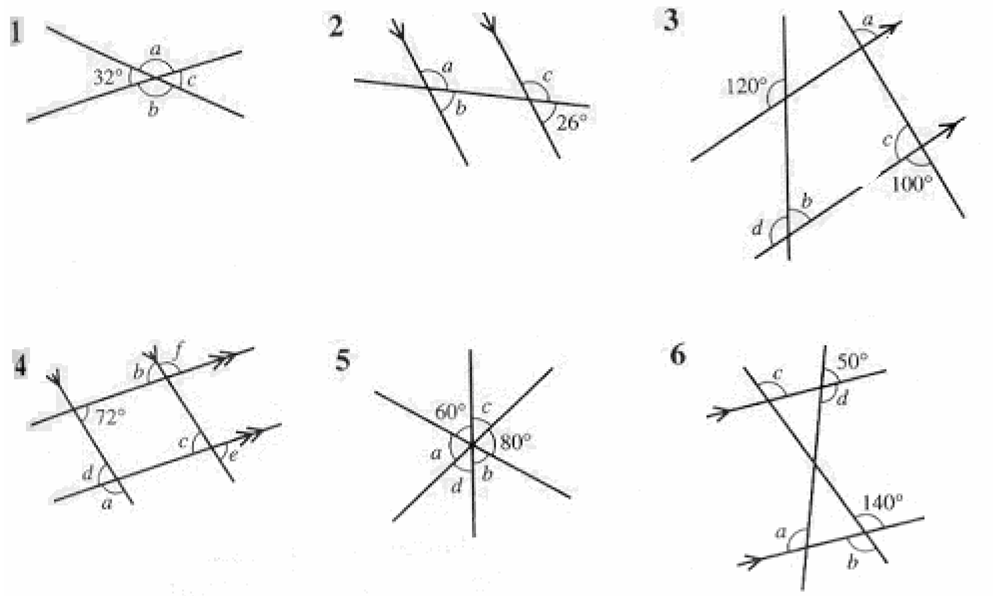


a = **b =** **c =** **d =** **f =** **g =**

h = **i =** **j =** **k =** **l =** **m =**

n =

b) Find the unknown angles:



1. a = **b =** **c =**

2. a = **b =** **c =**

3. a = **b =** **c =** **d =**

4. a = **b =** **c =** **d =** **e =** **f =**

g =

5. a = **b =** **c =** **d =**

6. a = **b =** **c =** **d =**

6 Operations with angles

The subunits of the degree are the **minute** of arc and the **second** of arc.

One minute $1' = \frac{1}{60}$ of a degree, that is $1^{\circ} = 60'$

One second $1'' = \frac{1}{60}$ of a minute, that is $1' = 60''$

These are the sexagesimal units, so an angle "a" can be expressed for example $a = 25^{\circ}\ 23'\ 40''$ and we need to operate angles expressed in this form.

6.1 Addition.

We need to add separately degrees minutes and seconds and then convert the seconds into minutes and the minutes into degrees if we get more than 60 subunits.

Example

Add $45^{\circ}\ 53'\ 40'' + 12^{\circ}\ 33'\ 35''$

Adding separately we get $45^{\circ}\ 53'\ 40'' + 12^{\circ}\ 33'\ 35'' = 57^{\circ}\ 86'\ 75''$ but $75'' = 1'\ 15''$ so we add $1'$ and get $87' = 1^{\circ}\ 27'$ we add 1° and finally the solution is $57^{\circ}\ 86'\ 75'' = 58^{\circ}\ 27'\ 15''$

6.2 Subtraction

We need to subtract separately degrees minutes and seconds, if we do not have enough seconds or minutes we convert one degree into minutes or a minute into seconds.

Example

Subtract $57^{\circ}\ 13'\ 21''$ and $12^{\circ}\ 43'\ 35''$ We write $57^{\circ}\ 13'\ 21''$ as $56^{\circ}\ 73'\ 21''$ and then:

$$\begin{array}{r} 56^{\circ}\ 72'\ 81'' \\ -\ \ 12^{\circ}\ 43'\ 35'' \\ \hline 44^{\circ}\ 29'\ 46'' \end{array}$$

Is the answer

6.3 Multiplication by a whole number

We multiply separately degrees minutes and seconds and then convert the seconds into minutes and the minutes into degrees when we get more than 60 subunits.

Example

Multiply $(22^{\circ}\, 13'\, 25'') \cdot 6$

22º	13’	25”
6	6	6
132	78	150
	80’	$\boxed{2'}$ 30”
133º	$\boxed{1^{\circ}}$ 20’	

Solution 133º 20’ 30”

6.4 Division by a whole number

We divide the degrees, and the remainder is converted into minutes that must be added to the previous, divide the minutes and we repeat the same that we have done before. The remainder is in seconds.

Example

Divide $(22^{\circ}\, 13'\, 25'') : 4$

22º 13’ 25” | 4

2º x 60 = 120

5º 33’ 21” quotient

133’

1’ x 60 = 60”

85”

1” remainder

Exercise 7

7.1 Add:

a) 28º 35’ 43” + 157º 54’ 21”

b) 49º 55’ 17” + 11º 5’ 47”

c) 233º 5' 59" + 79º 48' 40"

7.2

a) Subtract 34º 32' 12" – 11º 30' 22"

b) Calculate the complement of 13º 45' 12"

c) Calculate the supplement of 93º 30'

7.3 Given A = 22º 32' 41" Calculate:

a) 2·A

b) 3·A

c) $\frac{A}{5}$

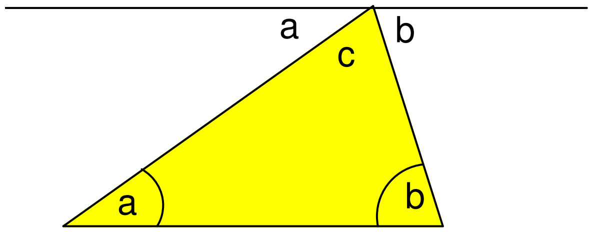
d) $\frac{2 \cdot A}{5}$

7 Angles in the polygons

7.1 Triangle

A triangle is a three-sided polygon.

The sum of the angles of a triangle is 180 degrees.

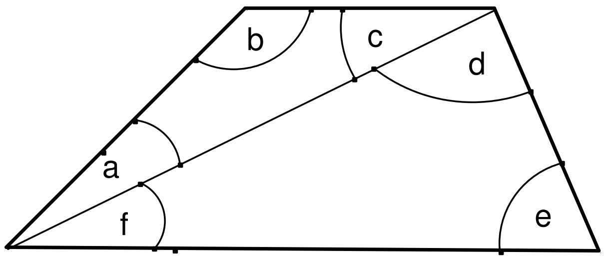


$$a + b + c = 180º$$

7.2 Quadrilateral

A quadrilateral is a four-sided polygon

The sum of the angles of any quadrilateral is 360º



$$a + b + c = 180º \qquad d + e + f = 180º$$

$$a + b + c + d + f = 360º$$

7.3 Polygon of n sides

The sum of the angles of a polygon with *n* sides, where *n* is 3 or more, is $(n-2)\cdot 180º$.

7.4 Regular Polygon

A regular polygon is a polygon whose sides are all the same length, and whose angles are all the same.

As the sum of the angles of a polygon with *n* sides is $(n-2)\cdot 180º$, each angle in a regular polygon is $\frac{(n-2)180º}{n}$.

Exercise 8 Complete the following table

Name	Number of sides	Sum of the interior angles	Name of the regular polygon	Interior angle
Triangle	3	180º	Equilateral triangle	
Quadrilateral	4	360	Square	
Pentagon			Regular pentagon	
Hexagon				
Heptagon			Regular heptagon	
	8			
Nonagon				
Decagon				
Undecagon				
Dodecagon				

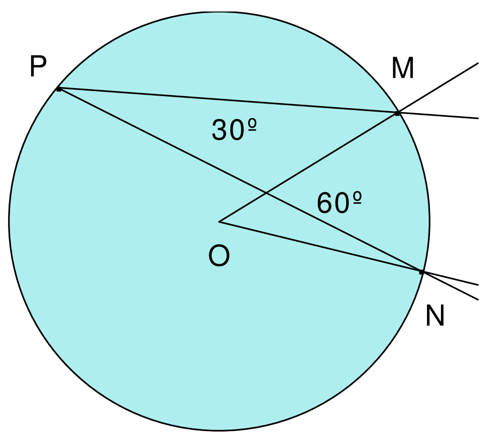
8 Angles in a circle

8.1 Central angle

A central angle is an angle whose vertex is the center of a circle and whose sides pass through a pair of points on the circle.

There is an arc of the circumference associated to the angle and it is, by definition equal to the central angle itself.

$\angle$MON is a central angle of 60º

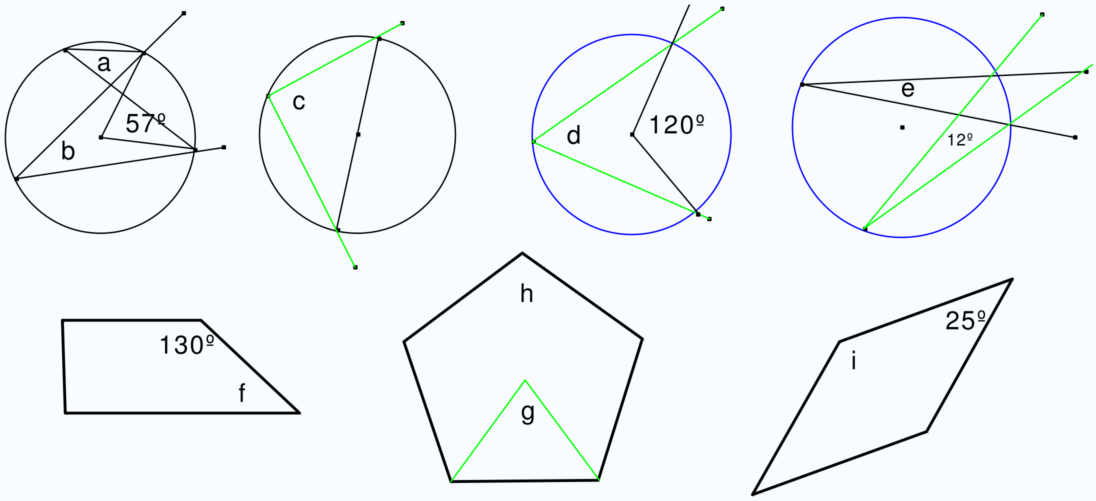


8.2 Inscribed angle

It is formed when two lines intersect a circle and its vertex is on the circumference. The measure of the intercepted arc in an inscribed angle is exactly the half of the central angle.

$\angle$MPN is an inscribed angle with the same arc, so it is a 30º angle.

Exercise 10 Calculate the angles in each figure and explain your answer:



a = **b =**

c = **d =**

f = **g =**

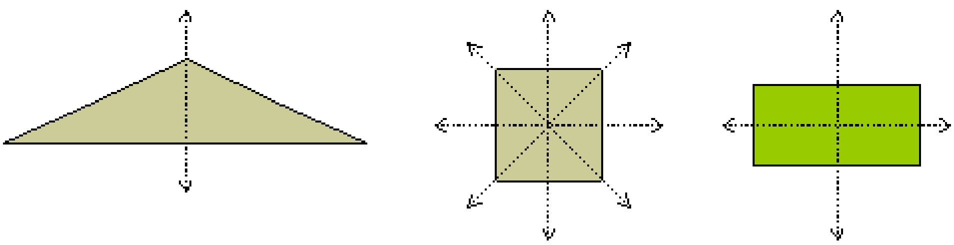
h = **i =**

9 Symmetric shapes

A figure that can be folded flat along a line so that the two halves match perfectly is a symmetric figure; such a line is called a line of symmetry.

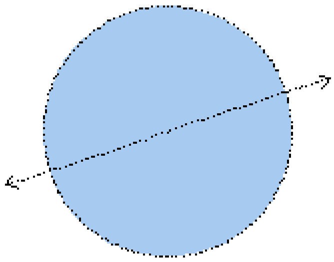
Examples:

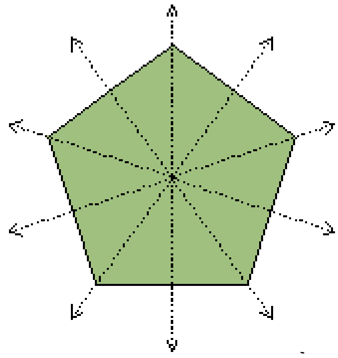
The triangle, the square and the rectangle below are symmetric figures. The dotted lines are the lines of symmetry.



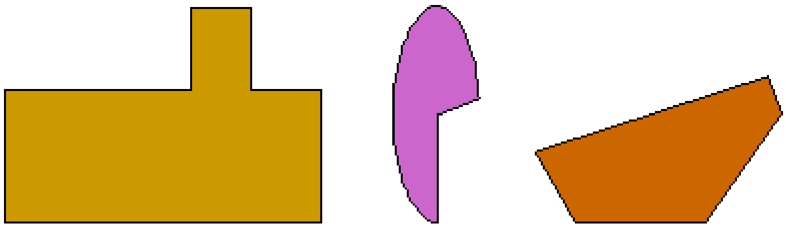
The regular pentagon below is a symmetric figure. It has five different lines of symmetry shown below.

The circle below is a symmetric figure. Any line that passes through its centre is a line of symmetry!



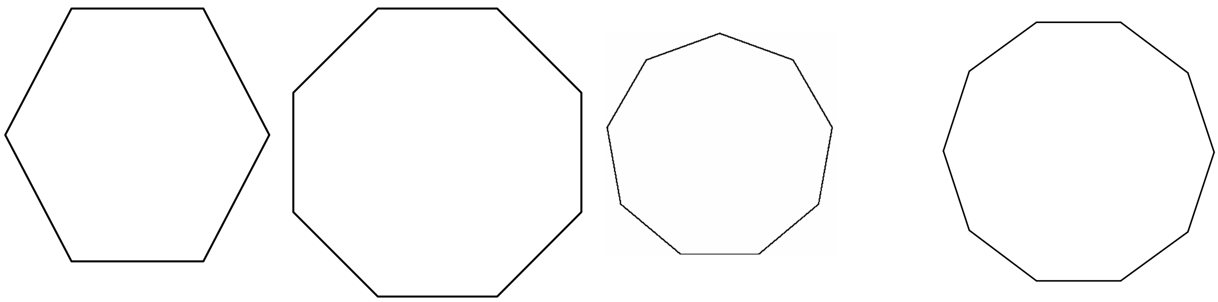


The figures shown below are not symmetric.



Exercise 11 Draw in all the lines of symmetry in the following regular polygons and use your answers to complete the table:

Polygon	Hexagon	Octagon	Nonagon	Decagon
Nº symmetry lines				



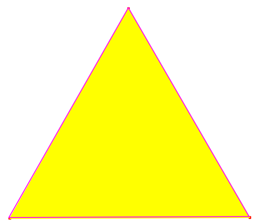
11 Polygons and 3-D shapes

1 Triangles

A triangle is a three-sided polygon. Sides of a polygon are also called edges.

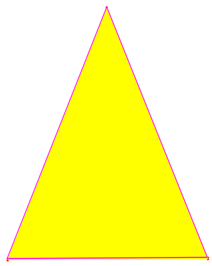
Triangles can be classified either by sides or by angles.

By sides:

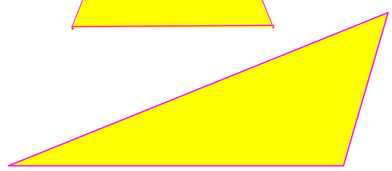


Equilateral Triangle

It is a triangle that has:
Three equal sides
Three equal angles of 60 degrees.

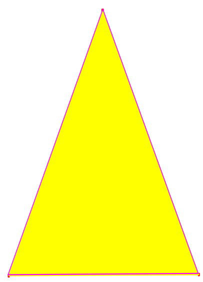


Isosceles Triangle
It is a triangle that has:
Two sides of equal length.
Two equal angles

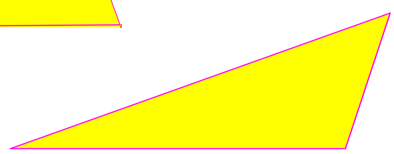


Scalene Triangle
It is a triangle that has:
Three sides of different lengths.
Three different angles

By angles:

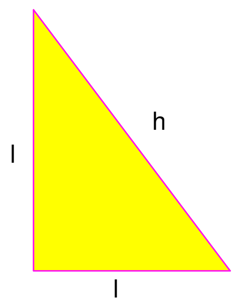


Acute Triangle
It is a triangle that has three acute angles.



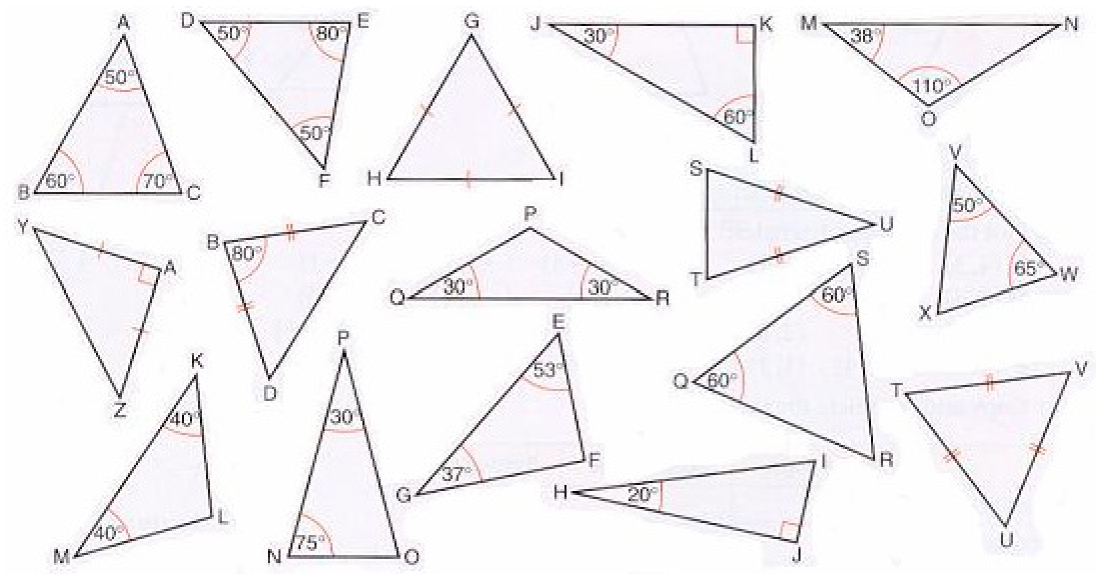
Obtuse Triangle
It is a triangle that has an obtuse angle, (measures more than 90º).

Right Triangle
It is a triangle that has a right angle.



The side opposite the right angle is called the **hypotenuse**. The two sides that form the right angle are called the **legs**.

Exercise 1 Classify the following triangles by their sides and by their angles. Find the asked angles of the triangles.



Triangle	By the sides	By the angles	Calculate
ABC			
DEF			
HGI			$\hat{H} =$
JKL			
MNO			$\hat{N} =$
YAZ			$\hat{Y} =$ $\hat{Z} =$
BCD			$\hat{C} =$ $\hat{D} =$
QPR			$\hat{P} =$
SUT			
XVW			$\hat{X} =$
MKL			$\hat{L} =$
NPO			$\hat{O} =$
GEF			$\hat{F} =$
QSR			$\hat{R} =$
UTV			$\hat{U} = \hat{T} = \hat{V} =$
HI			$\hat{I} =$

2 Equality of triangles

Two triangles are equal if the angles and the sides of one of them are equal to the corresponding angles and sides of the other

But we can assure that two triangles are equal if:

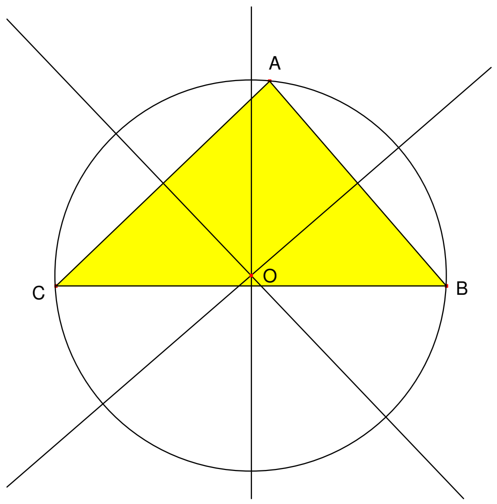
I. The lengths of their corresponding sides are equal
II. Two corresponding sides of two triangles and their included angles are equal.
III. One side and the angles at any side are equal.

3 Points and lines associated with a triangle

3.1 Perpendicular bisectors, circumcentre

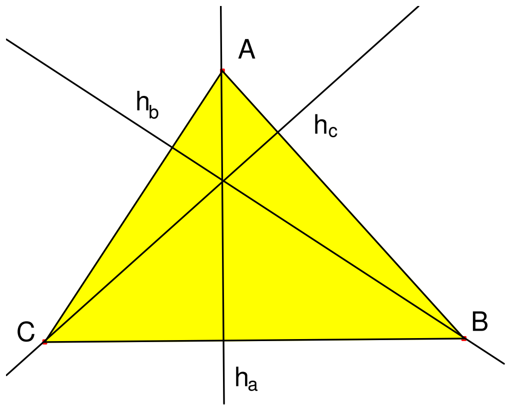
A perpendicular bisector of a triangle is a straight line passing through the midpoint of a side and perpendicular to it.

The three perpendicular bisectors meet in a single point, it is called the triangle's circumcentre; this point is the centre of the circumcircle, the circle passing through the three vertices.

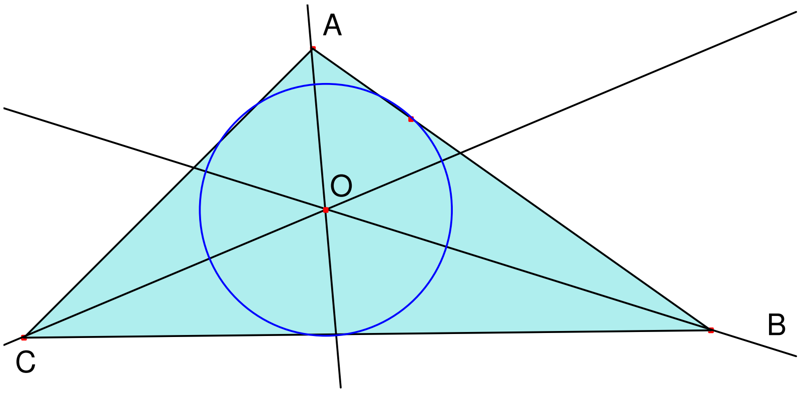


3.2 Altitudes, orthocentre

An altitude of a triangle is a straight line through a vertex and perpendicular to the opposite side. This opposite side is called the *base* of the altitude, and the point where the altitude intersects the base (or its extension) is called the *foot* of the altitude.
The length of the altitude is the distance between the base and the vertex. The three altitudes intersect in a single point, called the orthocentre of the triangle.



3.3 Angle bisectors, incentre



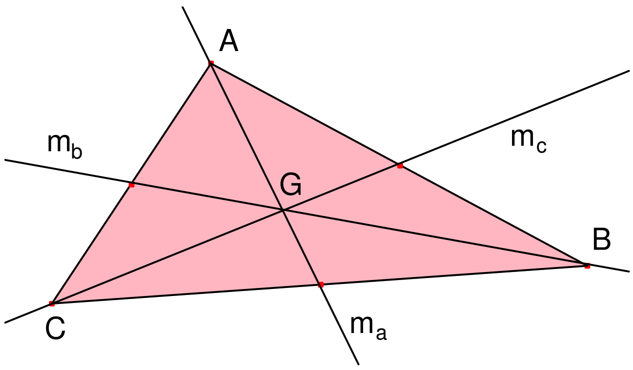
An angle bisector of a triangle is a straight line through a vertex, which cuts the corresponding angle in half. The three angle bisectors intersect in a single point called the incentre, which is the centre of the triangle's incircle.
The incircle is the circle, which lies inside the triangle and is tangent to the three sides.

3.4 Medians, barycentre

A median of a triangle is a straight line through a vertex and the midpoint of the opposite side.

The three medians intersect in a single point, the triangle's barycentre.

This is also the triangle's centre of gravity.



Exercise 2

a) Construct a triangle with a base of 6 cm and angles of 60º and 45º.

b) Draw the three perpendicular bisectors and check that the three of them have a common point. Which is its name?

c) Use a compass to draw the circumcircle.

Exercise 3

a) **Construct a triangle with a base of 5 cm, one side 7 cm and the angle between these sides of 50º.**

b) **Draw the three altitudes and check that the three of them have a common point. Which is its name?**

Exercise 4

a) **Construct a triangle with sides of 9.5 cm, 7.2 cm and 6 cm.**

b) **Draw the three angle bisectors and check that the three of them have a common point. Which is its name?**

c) **Draw the incircle of the triangle**

Exercise 5

a) Construct a triangle with sides of 6.4 cm, 7.8 cm and the angle between these sides of 37º.

b) Draw the three medians and check that the three of them have a common point. Which is its name?

c) Check that the barycentre in every median is at a distance of the vertex double than the one it has to the midpoint of the side.

Exercise 6 Draw in your notebook each of these triangles and find the asked point.

Triangle	DATA (lengths in cm)	POINT
ABC	a = 5 cm, b = 3 cm, c = 6 cm	Orthocentre
DEF	d = 6 cm, $\hat{E} = 33º$, $\hat{F} = 52º$	Barycentre
GHI	h = 7 cm $\hat{H} = 53º$ $\hat{I} = 67º$	Incentre
JKL	k = 4 cm, j = 6 cm $\hat{L} = 42º$	Circumcentre
MNO	m = 8 cm n = 6.6 cm $\hat{M} = 62º$	Circumcentre
PQR	p = 6 cm, q = 8 cm, r = 10 cm	Orthocentre and circumcentre

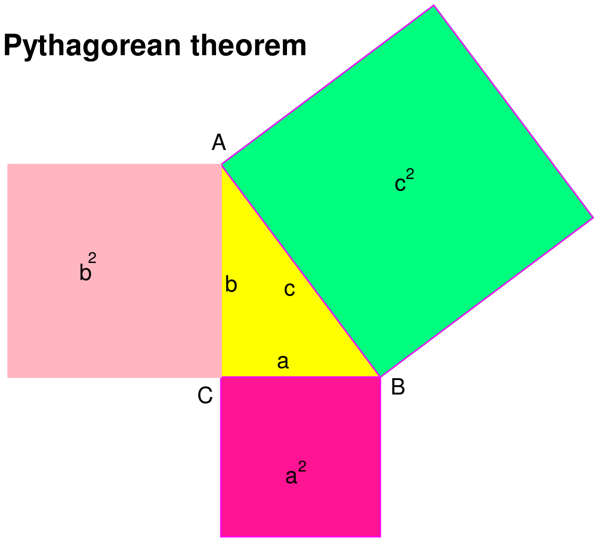
4 The Pythagorean Theorem

In a right triangle one of the angles of the triangle measures 90 degrees. The side opposite the right angle is called the hypotenuse. The two sides that form the right angle are called the legs.

In a right triangle the sum of the squares of the lengths of the legs equals the square of the length of the hypotenuse. This is known as the Pythagorean Theorem.

For the right triangle in the figure, the lengths of the legs are *a* and *b*, and the hypotenuse has length *c*. Using the Pythagorean Theorem, we can write that $a^2 + b^2 = c^2$.

The acute angles of a right triangle are complementary.



Exercise 7 Find the value of the third side of the following right triangles ABC in which $\hat{A} = 90º$. Round the calculations to the nearest hundredth:

a) **b = 7 cm, a = 9 cm.**

b) **b = 3 cm , c = 4 cm**

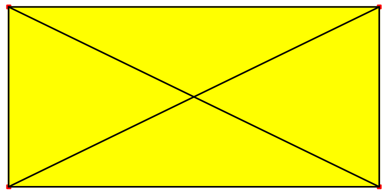
c) **a = 12.5 cm, c = 8.6 cm**

d) **a = 0.34 cm, c = 0. 27 cm**

5 Quadrilaterals

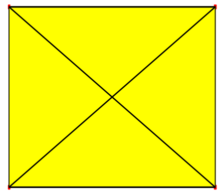
A quadrilateral is a polygon with four sides. The four angles of any quadrilateral add up to 360º.

These are the special quadrilaterals



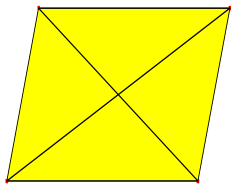
- Rectangle

All the angles are right angles.
Opposite sides are equal.
Diagonals have the same length and bisect each other.



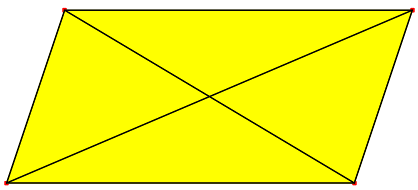
- Square

All the angles are of 90º
All the sides are equal in length
Diagonals have the same length and bisect each other at right angles.



- Rhombus

All the sides have the same length.
Opposite sides are parallel.
Opposite angles are equal.
Diagonals bisect each other at right angles.

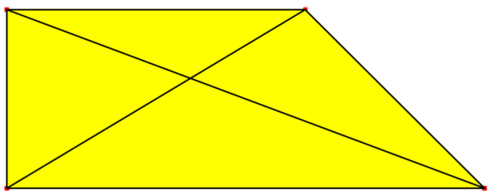


- Parallelogram or rhomboid

Opposite sides are parallel and have the same length.
Opposite angles are equal.
Diagonals have the same length and bisect each other.

- Trapezium (U.K.) or

Trapezoid (U.S.A.)



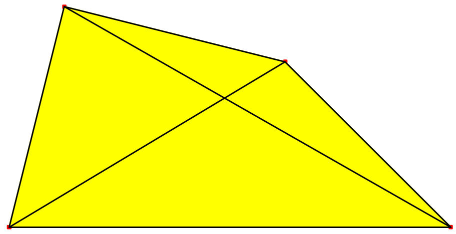
Trapezium (U.K.)

One pair of sides is parallel. The two sides that are parallel are called the bases of the trapezium.

In an isosceles trapezium the base angles are equal, and so is the pair of non-parallel sides.

- Trapezoid (U.K.) or Trapezium (U.S.A.)

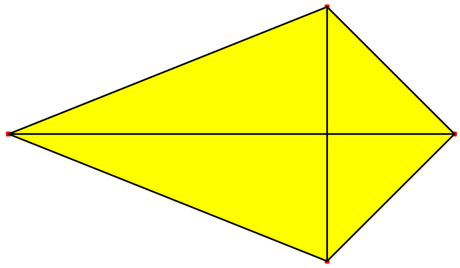
Quadrilateral that has no parallel sides



Trapezoid (U.K.)

- Kite

It is a special type of trapezoid in which:
The adjacent sides are equal.
Diagonals intersect each other at right angles



Exercise 8 One of the sides of a rectangle measures 4 cm and the diagonal 6 cm

a) Construct the rectangle.
b) Calculate the other side and the perimeter.

Exercise 9 In a rectangle the length of its sides are 8.3 cm and 5.4 cm. Calculate the length of its diagonal.

Exercise 10 Construct a rhombus with diagonals 4 cm and 7 cm. Calculate the perimeter.

Exercise 11 Construct a square with a diagonal of 7.5 cm. Calculate the perimeter.

Exercise 12 Say how many axes of symmetry there are in:

a) A square

b) A rectangle

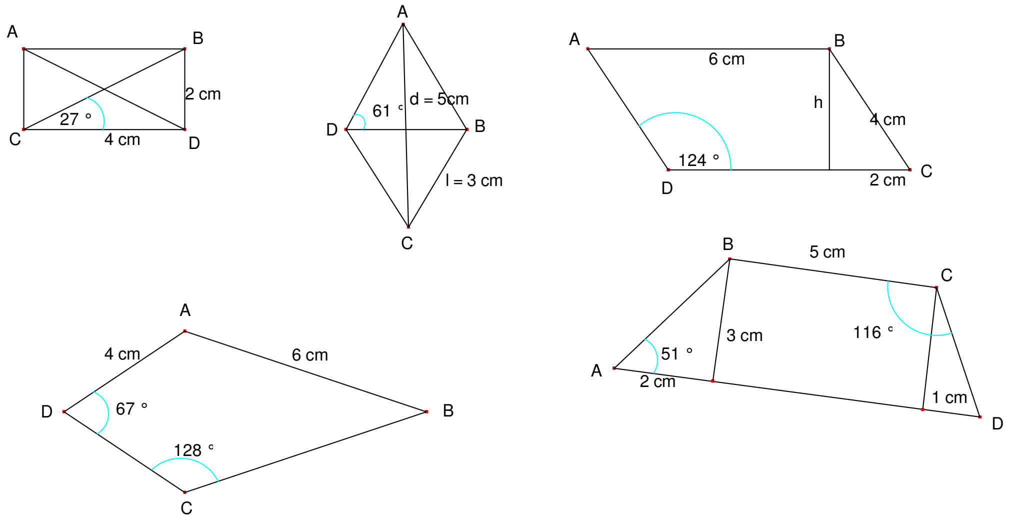
c) A rhombus

d) A trapezium

e) A pentagon

f) **An hexagon**

Exercise 13 Find all the missing sides and angles in the polygons below.

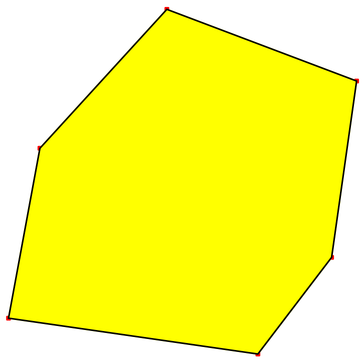


6 Regular polygons

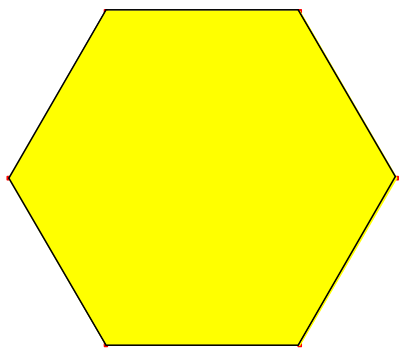
6.1 Definition and names

A **regular polygon** is a polygon in which all angles are equal and all sides have the same length.

Others polygons are called **irregular polygons**



Irregular hexagon



Regular Hexagon

A five-sided polygon is a **Pentagon**

A six-sided polygon is a **Hexagon**

A seven-sided polygon is a **Heptagon**

An eight-sided polygon is an **Octagon**

A nine-sided polygon is a **Nonagon**

A ten-sided polygon is a **decagon**

With more than ten sides it is better to call them a polygon of **"n"** sides.

6.2 Proprieties of the regular polygons

All the regular polygons can be circumscribed by a circle, this is called the **circumcircle**, and it is a circle which contains all the vertices of the polygon. The centre of this circle is called the **circumcenter** and it is the **centre** of the regular polygon. The radius of this circle is also the **radius** of the polygon.

Apothem is the line drawn from the centre of the polygon perpendicular to a side.

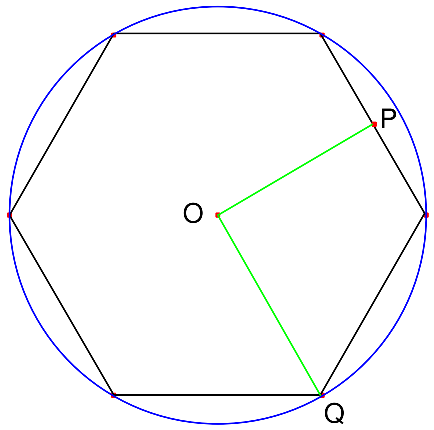
Example:

In this hexagon

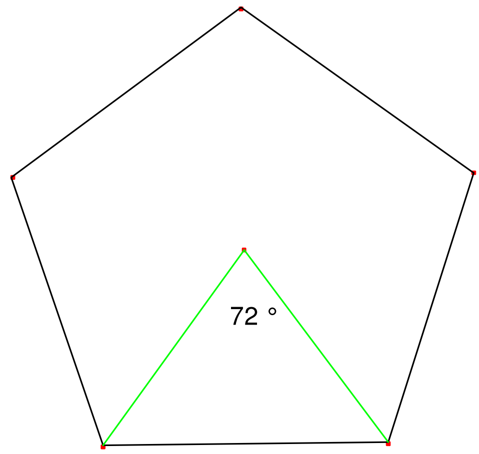
O is the centre

OP is the apothem

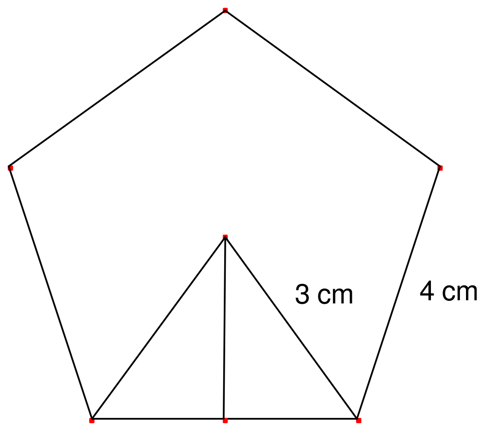
OQ is the radius



Central angle is the angle formed with two radiuses drawn in two consecutive vertices, it is equal to 360º divided by the number of sides



Exercise 14 Calculate the apothem of this pentagon



Exercise 15 Construct a regular hexagon with a radius of 3 cm. Calculate the apothem and the central angle

7 Circle and circumference

7.1 Definitions

- A circumference is the collection of points in a plane that are all at the same distance from a fixed point. The fixed point is called the **centre**.

- A circle is the shape inside the circumference.

- Circle is commonly used meaning circumference

7.2 Lines in a circle

1. **Radius** is the distance from the centre to the edge. (Segment OP)

2. **Diameter** is the segment between two points of the circle that passes through the centre. (Segment QR)

3. **Chord** is a straight line between two points of the circumference. (Segment CD)

4. **Arc** is a part of the circumference of a circle. (Curve CD)

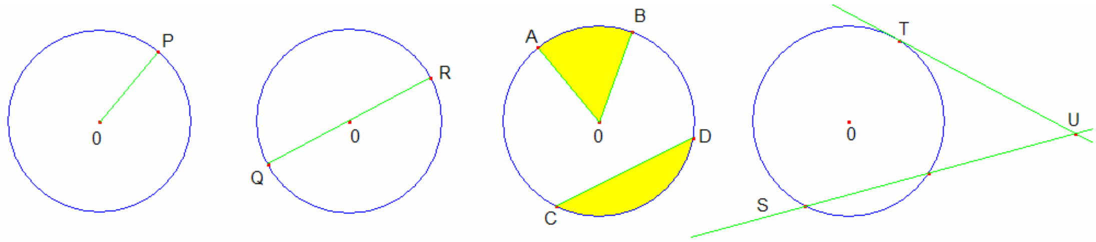
5. **Sector** is the shape made by two radiuses (radii) of the circle. (OAB)

6. **Segment** is a shape made by a chord and an arc. (Curve CD and segment CD)

7. **Tangent** is a straight line that touches the circle at only one point. (TU)

A tangent of a circle is always perpendicular to a correspondent radius

8. **Secant** is a line that intersects two points of a circle. (SU)



7.3 Positions of circles

- **Concentric circles** are circles, which have the same centre
- **Eccentric circles** have different centres.
- **Interior circles** are circles, one inside the other. They can be concentric or eccentric circles.
- **Exterior circles** have their centres at a distance greater or equal to the sum of their radiuses.
- **Tangent circles** have a common point. They can be interior or exterior circles.
- **Secant circles** have two common points.

Exercise 16 Draw circles in your notebook in all the different positions that you can see in 7.3

Exercise 17

a) Construct a circumference with a radius of 5 cm and a chord of 7cm.

b) b) Calculate the distance from the centre to the chord.

c) Draw two secants and one tangent to the circle.

3-D shapes.

8 Polyhedrons

Polyhedrons are geometric solids whose faces are formed by polygons

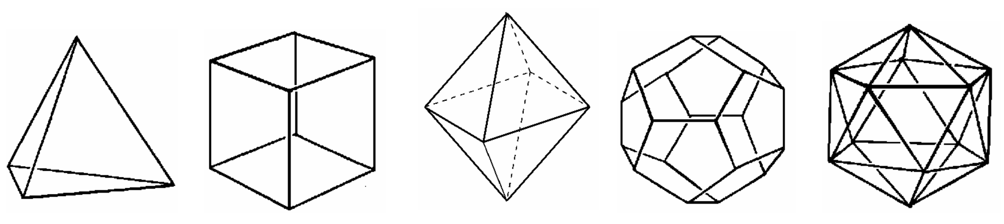
Components:

- **Faces** are the polygons that bound the polyhedron
- **Edges** are the lines where two faces join.
- **Vertices** are the points where three or more edges meet
- **Diagonal** is a segment that joins two non-consecutive vertices
- **Dihedron angle** is the angle between two faces

8.1 Regular polyhedrons.

The **regular polyhedrons** have all their faces formed by identical regular polygons. They are:

Tetrahedron: Four equal faces, each of them is a equilateral triangle
Cube: Six squares
Octahedron: Eight equilateral triangles
Dodecahedron: twelve regular pentagons
Icosahedron: Twenty equilateral triangles

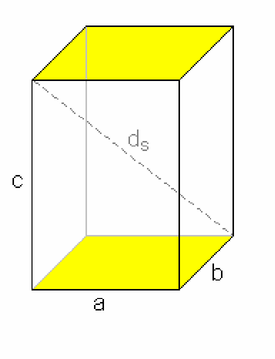


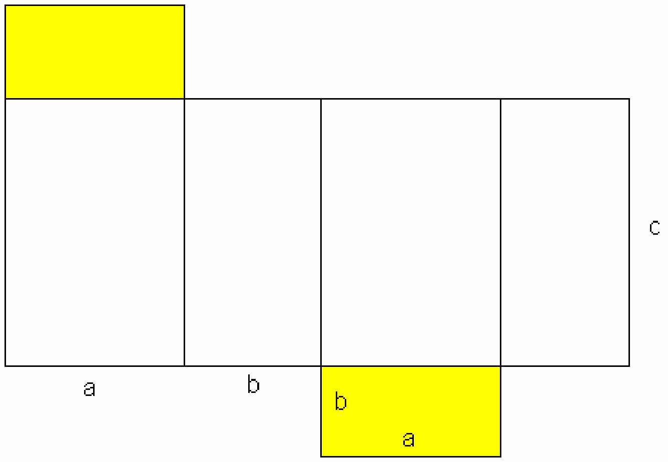
Tetrahedron Cube Octahedron Dodecahedron Icosahedron

8.2 Cuboids

A cuboid is a geometric objet with faces that are rectangles (in some cases squares). The special case in which all the faces are squares is the **cube.**

They have 6 faces, 12 edges and 8 vertices





8.3 Prisms

A prism is a polyhedron with two equal and parallel faces that are polygons (bases) and the other faces are parallelograms.

The distance between the two bases is the **height** of the prism.

Prisms can be right prisms when the parallelogram faces are perpendicular to the bases, otherwise they are oblique prisms.

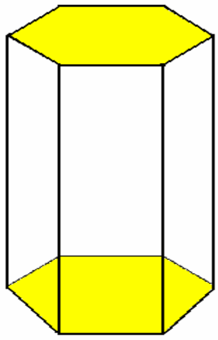
Depending on the polygons of the bases they can be:

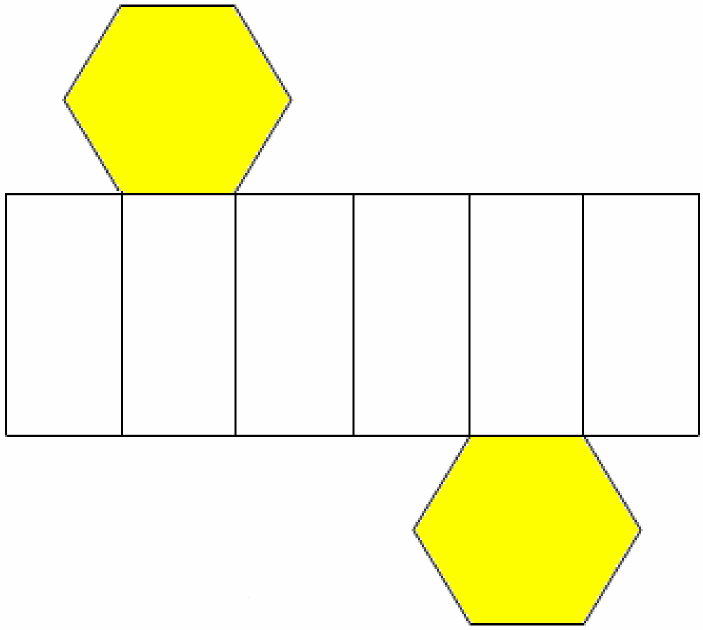
Triangular prism, with triangular bases

Square prism, with square bases (these are also called cuboids or parallelepipeds),

Pentagonal prism, when the bases are pentagons.

Hexagonal prism, etc.





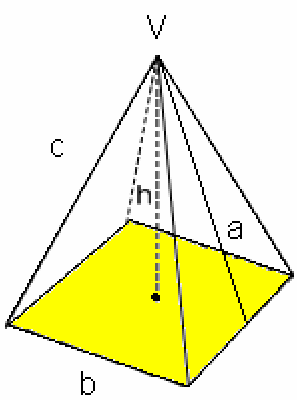
Hexagonal prism

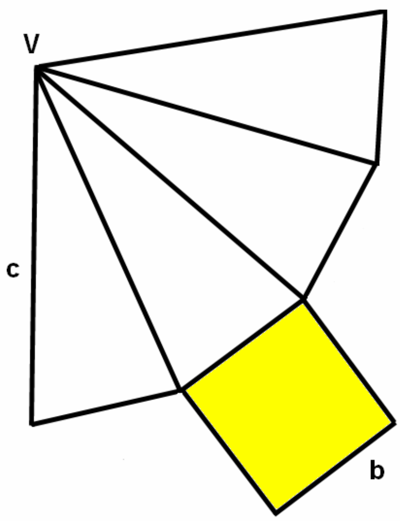
8.4 Pyramids

A pyramid is a polyhedron in which one of its faces is any polygon called the **base** and the other faces are triangles that join in a point that is the **apex.**

The height of a pyramid (h) is the distance from the base to the apex.

Like the prisms, pyramids can also be right or oblique and depending on the polygons of the base they can be: **triangular, square, pentagonal, hexagonal,** etc.





Square pyramid

8.5 Euler formula

The Euler formula relates the number of vertices *V*, edges *E*, and faces *F* of any polyhedron:

$F + V = E + 2$

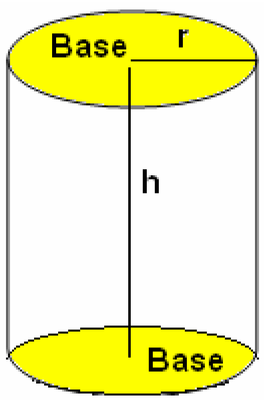
Exercise18. Complete this table and check in each case that the Euler formula comes true.

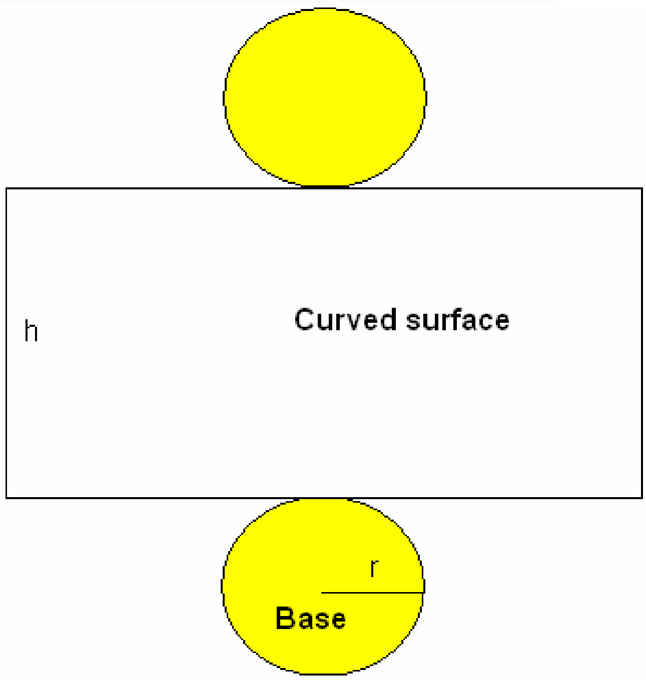
NAME OF THE SOLID	Number of faces	Number of vertices	Number of edges
Cuboid			
Tetrahedron			
Cube			
Octahedron			
Dodecahedron			
Triangular prism			
Pentagonal prism			
Hexagonal pyramid			
Octagonal pyramid			

9 Solids of revolution

9.1 Cylinders

A **cylinder** is a curvilinear geometric solid formed by a **curved surface** with all the points at a fixed distance from a straight line that is the **axis** of the cylinder and by two circles perpendicular to the axis that are the **bases**.

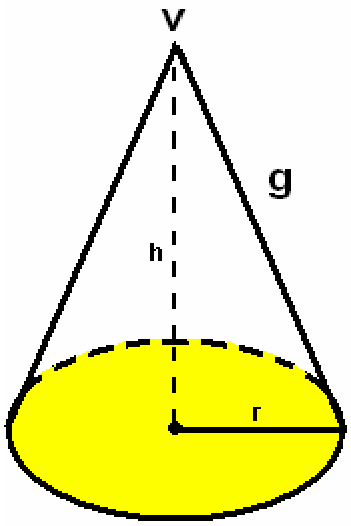


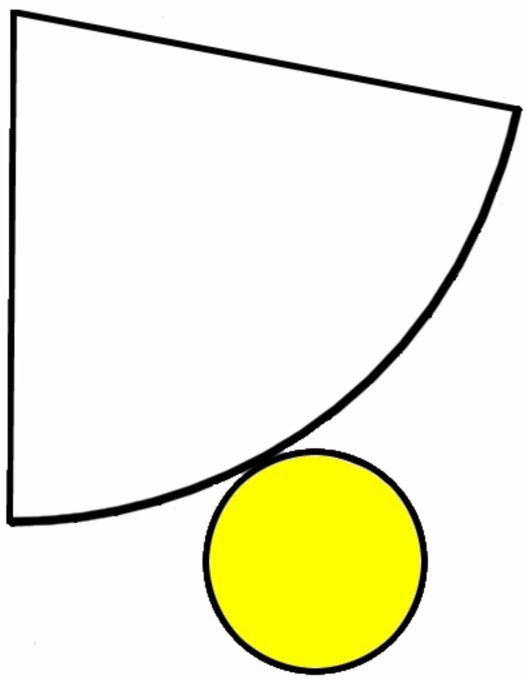


The height of a cylinder (h) is the distance from the base to the top.
The pattern of the curved surface when it is unrolled is a rectangle
The two bases are circles; the radius of the cylinder is the radii of the bases.

9.2 Cones

A cone is a solid bounded by a curved surface that has a common point (vertex), with a line that is the axis of the cone and a circle perpendicular to the axis that is called the base of the cone.

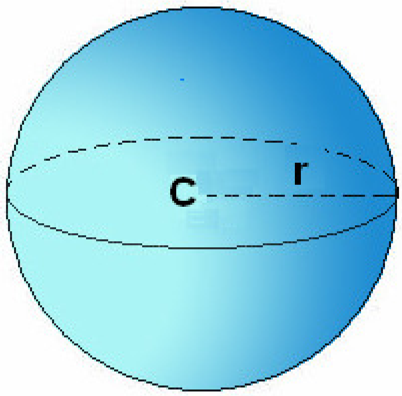




Vertex or apex is the top of the cone (**V**).

Generatrix of the cone is the straight line that joins the vertex with the circle of the base (**g**).

9.3 Sphere



A sphere is the solid bounded by a surface in which all points are at the same distance **r** from a fixed point that is the **centre** of the sphere **C**.

The distance from the centre to the surface of the sphere is called the radius of the sphere **r**

12 Areas

1 Area

The area of a figure measures the size of the region enclosed by the figure. This is expressed in one of the square units, square meters, square centimetres, square inches, and so on.

2 Quadrilaterals

2.1 Area of a square

If 'a' is the length of the side of a square, the area is $a \times a$ or a^2.

Example:

What is the area of a square if the side-length is 7cm?

The area is 7x7 =49 cm^2

2.2 Area of a rectangle

The area of a rectangle is the product of its width and length.

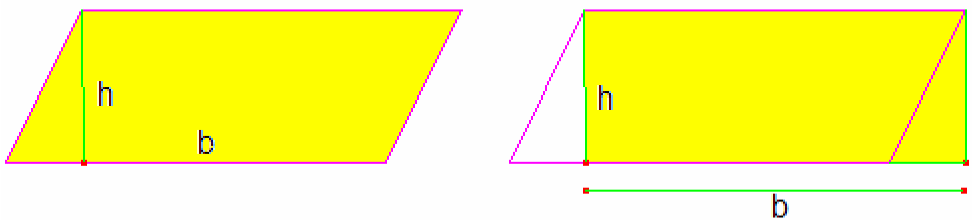
Example:

What is the area of a rectangle having a length of 12 m and a width of 3.2 m?

The area is the product of these two side-lengths, which is $12 \times 3.2 = 38.4$ m^2

2.3 Area of a parallelogram

The area of a parallelogram is $b \times h$, where b is the length of the base of the parallelogram, and h is the perpendicular height. To picture this, consider the parallelogram below:



We can cut a triangle from one side and paste it to the other side to form a rectangle with side-lengths b and h. the area of this rectangle is b × h.

Example:

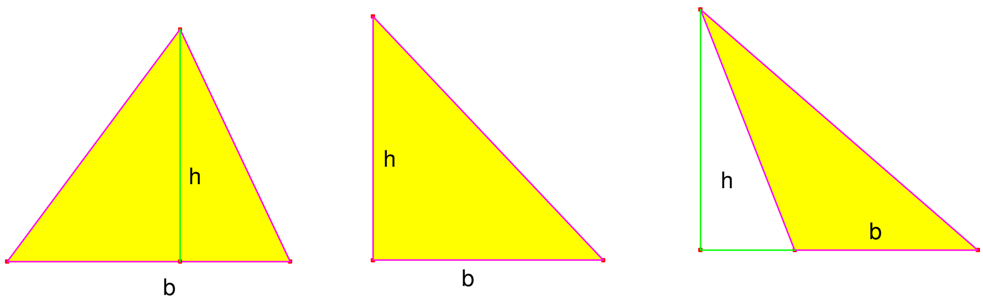
What is the area of a parallelogram with a base of 82 mm and a perpendicular height of 73 mm?

The area $A = 82 \times 73 = 5986 \text{ mm}^2$

3 Area of a triangle

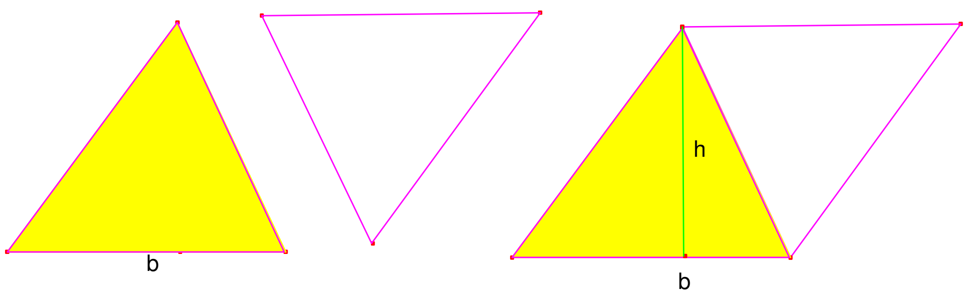
Consider a triangle with base length *b* and height *h*.

The area of the triangle is $\frac{1}{2} b \times h$.



The perpendicular height can be inside the triangle, one of its sides or outside of the triangle as can be seen in the picture.

To demonstrate this formula, we could take a second triangle identical to the first, then rotate it and "paste" it to the first triangle as pictured below:

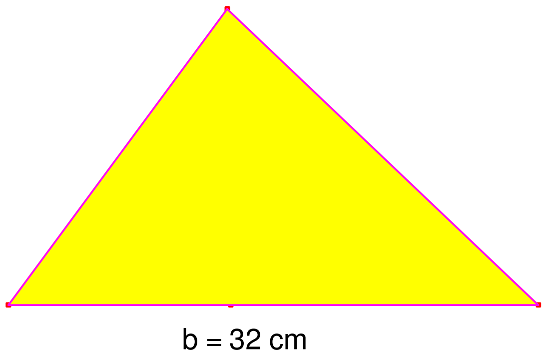


The figure formed is a parallelogram with base length *b* and height *h*, and has area $b \times h$.

This area is twice that the area of the triangle, so the area of the triangle is $\frac{1}{2} b \times h$

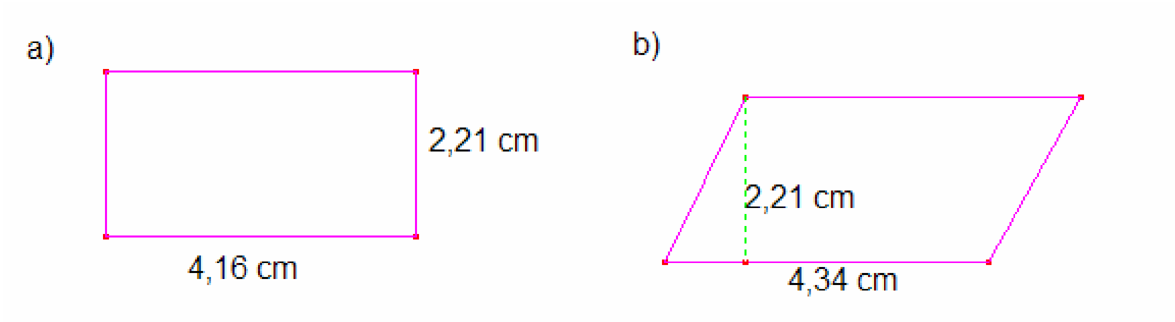
Example:

What is the area of the triangle of the picture on the right?

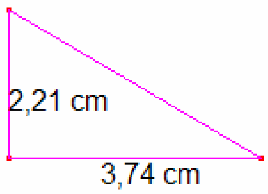


The area of a triangle is half the product of its base and height, which is $A = \frac{1}{2} 15 \times 32 = 240 \text{ cm}^2$

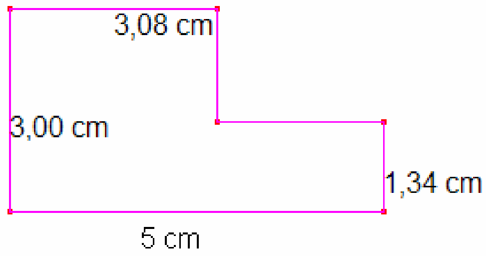
Exercise 1 Calculate the area of these shapes



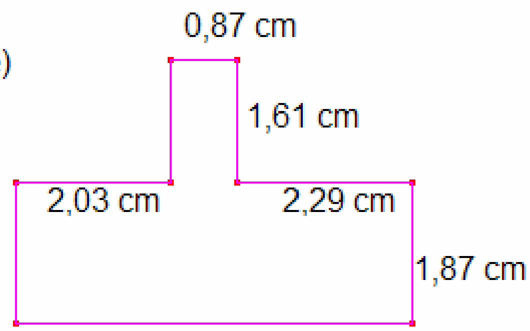
c)


2,21 cm
3,74 cm

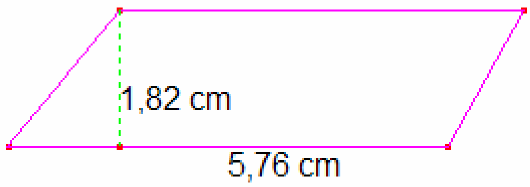
d)


3,08 cm
3,00 cm
1,34 cm
5 cm

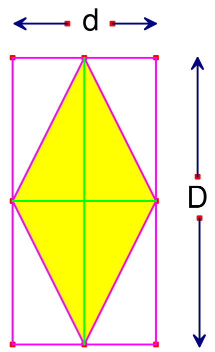
e)


0,87 cm
1,61 cm
2,03 cm
2,29 cm
1,87 cm

f)


1,82 cm
5,76 cm

4 Area of a rhombus



The area of a rhombus can be calculated as the area of a parallelogram as we have seen in point 2.3, but it can also be calculated if we know the length of the diagonals.

The area of the rectangle is $D \times d$, there are four equal triangles inside and outside the rhombus, so the area of the rhombus is $A = \dfrac{D \times d}{2}$.

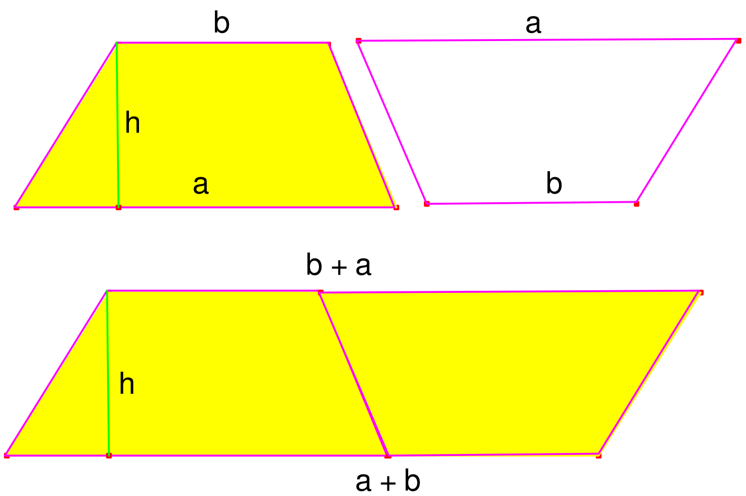
The perimeter of a rhombus is $4l$ if l is the length of each side.

Exercise 2 Calculate the area and the perimeter of a rhombus, whose diagonals are of 6cm and 8 cm.

5 Area of a trapezium

If a and b are the lengths of the two parallel bases of a trapezium, and h is its height, the area is $A = \dfrac{a+b}{2} \cdot h$.

To demonstrate this, consider two identical trapezoids, and "turn" one around and "paste" it to the other along one side as it is drawn below:



The figure formed is a parallelogram having an area of $A = (a+b)\cdot h$, which is twice the area of one trapezium.

E*xample:*

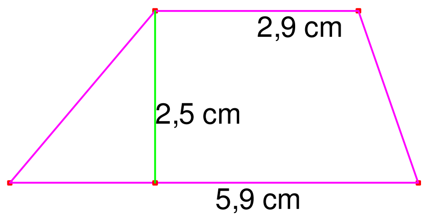
What is the area of a trapezium having bases 13 and 9 and a height of 6?

Using the formula for the area of a trapezoid, we see that the area is

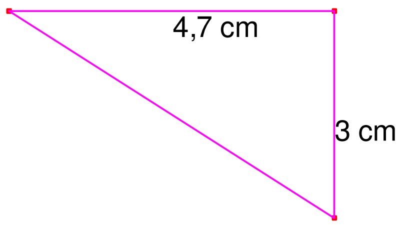
$\frac{13+9}{2}\times 6 = 66$ units of area

Exercise 3 Calculate the area of these shapes

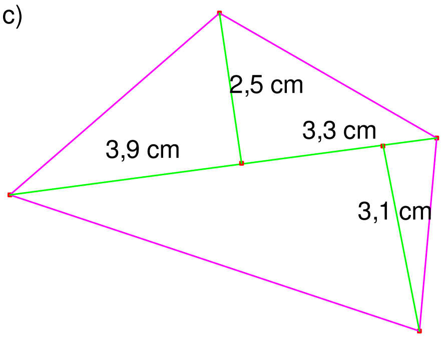
a)



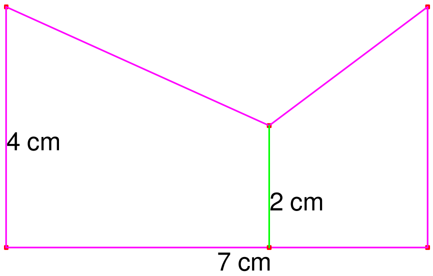
b)



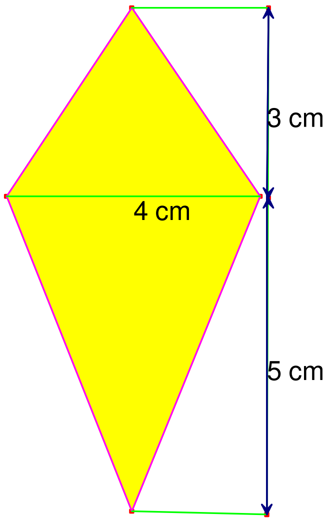
c)


2,5 cm
3,3 cm
3,9 cm
3,1 cm

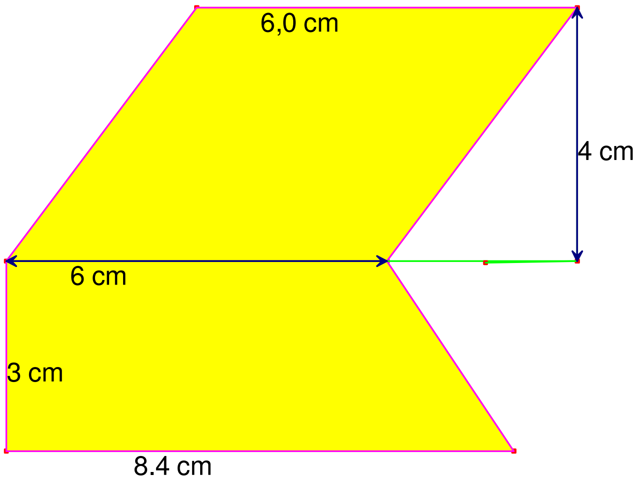
d)


4 cm
2 cm
7 cm

e)


3 cm
4 cm
5 cm

f)

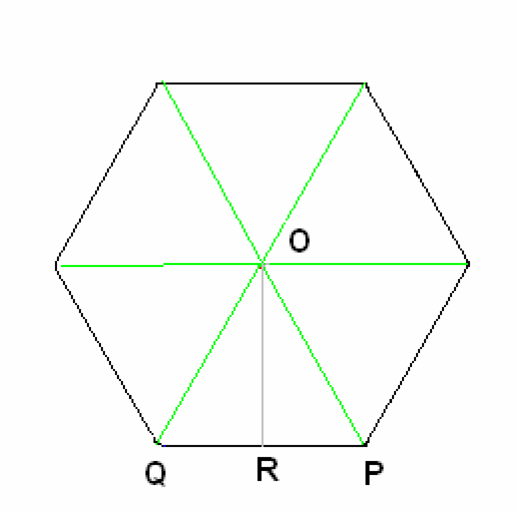

6,0 cm
4 cm
6 cm
3 cm
8.4 cm

6 Measures in regular polygons

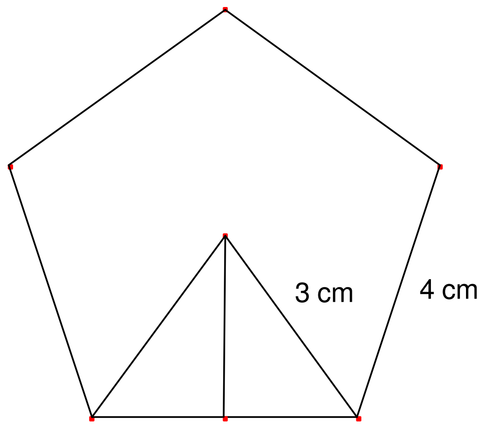
6.1 Area and perimeter of regular polygons

All the regular polygons can be divided into n equal isosceles triangles and in any of these triangles the base is the side of the polygon (like QP) and the height is the apothem OR, the area of each triangle is $A = \frac{\text{poligon side} \times \text{apotheme}}{2}$ so the area of a regular polygon with n sides of length l and apothem a is.

$A = \frac{l \times a}{2} n = \frac{p \times a}{2}$ Where p is the perimeter



Exercise 4 Calculate the apothem of this pentagon and the its area



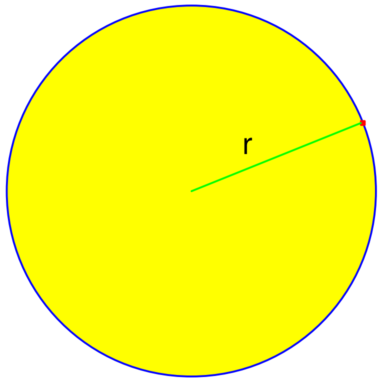
6 Measures in a circle

6.2 Area of a circle

The area of a circle is $\pi \times r^2$, where *r* is the length of its radius. π is a number that is approximately 3.14159.

Example:

What is the area of a circle having a radius of 53 cm, to the nearest tenth of a square cm? Using an approximation of 3.1415927 for π, and the fact that the area of a circle is $\pi \times r^2$, the area of this circle is $3.1415927 \times 53^2 = 8824.73$ square cm, which is 8824.7 cm^2 rounded to the nearest tenth.



6.3 Length of a circumference

The length of a circle's circumference is $l = 2\pi \cdot r$

Example:

The length of the circumference of the previous circle using for π 3.14 is $l = 3.14 \times 53 = 166.4cm$

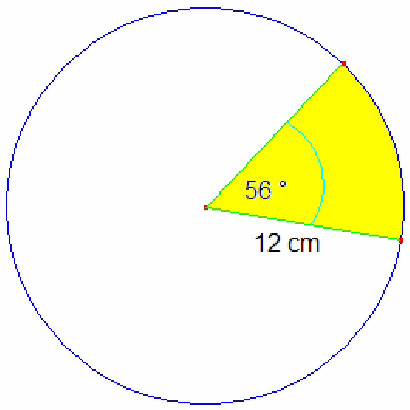
6.4 Length of an arc of the circumference

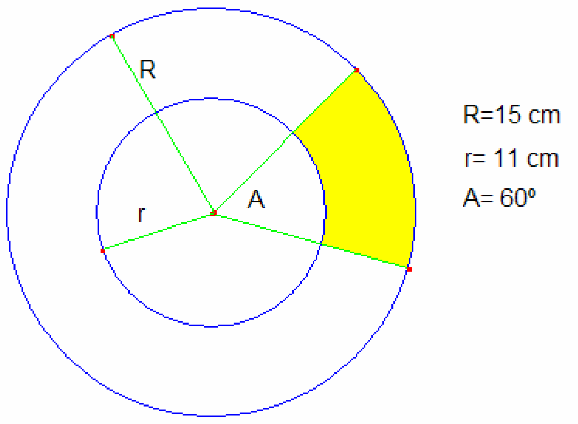
The length of an arc of the circumference of n degrees is $l = \frac{2\pi r}{360} n$ because there is a direct proportionality between the length of the arc and the corresponding angle.

6.5 Area of a sector

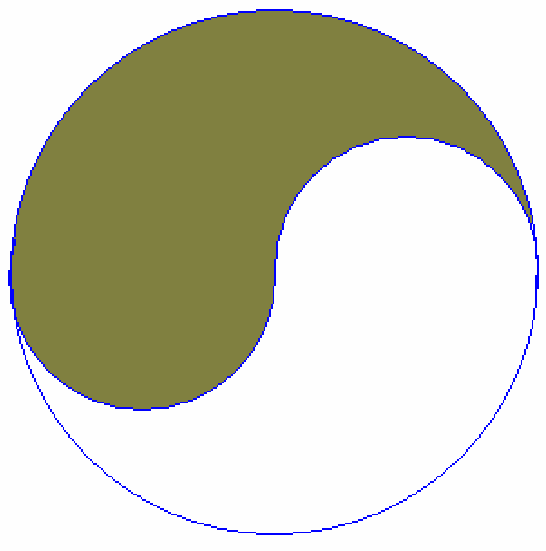
There is also a direct proportionality between the area of a sector and the corresponding angle, so $A = \frac{\pi \cdot r^2}{360} n$.

Exercise 5 Calculate the length or the arcs and the area of the shaded part in the pictures.

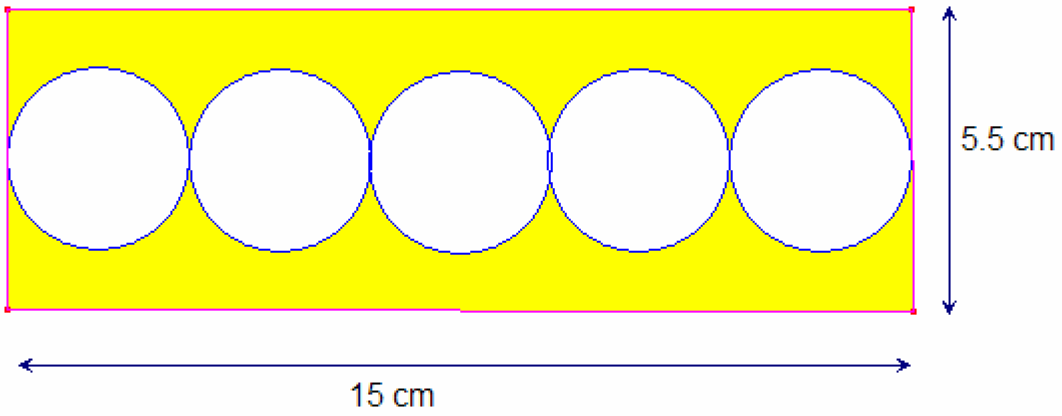


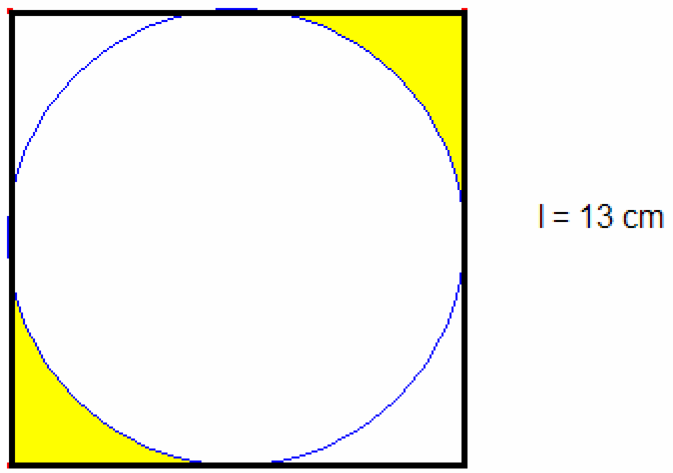


Exercise 6 Calculate the area of the shaded part in this logo.



Exercise 7 Calculate the area of the shaded part in these pictures.


5.5 cm
15 cm


l = 13 cm

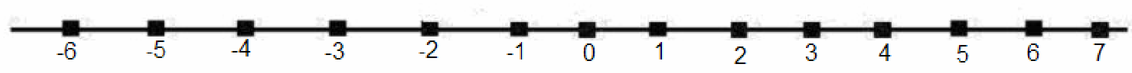
13 Graphs

Previous ideas

The number line

Numbers can be represented on a line in this way

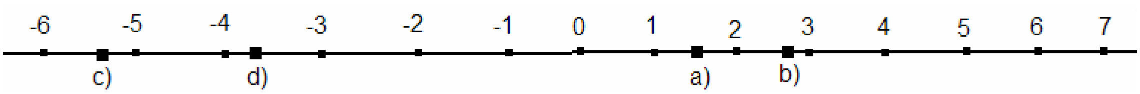
1. Draw a line.
2. Choose a point for zero.
3. Positive numbers are drawn to the right, negatives to the left.



Decimal numbers can also be placed on the line

Example 1 Place approximately on the number line the following numbers

a) 1.5 b) 2.8 c) - 5.3 d) - 3.7



1 Graphs

How to plot points on a plane

To represent points in a plane we use two perpendicular number lines.

The horizontal line is called the x-axis (positives numbers to the right and negatives to the left).

The vertical line is called the y-axis (positive numbers up and negatives down).

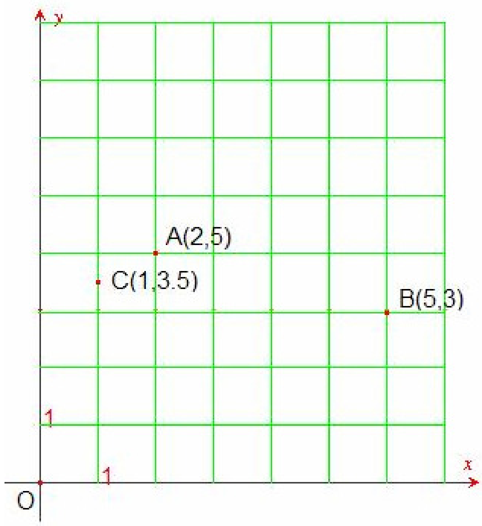
The common point of the two lines is called the origin O

For plotting a point we need an **ordered** pair of numbers, be careful! The order in which the couple is written is important.

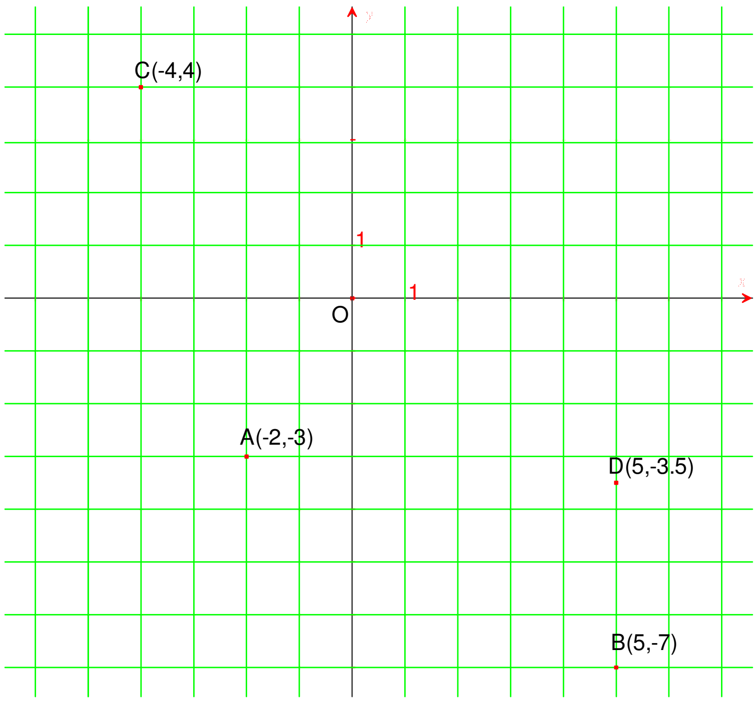
The firs number of the pair is the x-coordinate (abscissa).

The second one is the y-coordinate (ordinate).

Example 2 Plot the numbers A(2,5) B(5,3) and C(1,3.5)



Example 3 Plot the numbers A(-2,-3) B(5,-7) and C(-4,4) and D(5,-3.5)



Exercises

Exercise 1 In the graph data is plotted about the height and weight of a group of students, where x is the height in cm and y is the weight in kg of each one. Answer these questions.

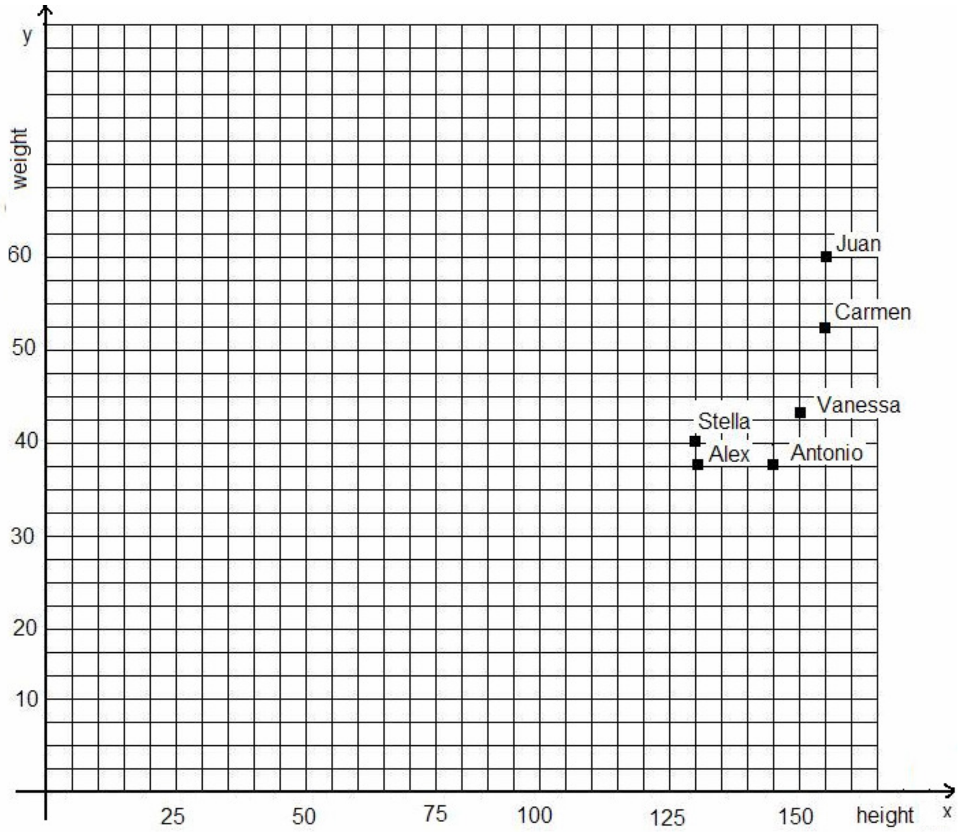
a) Who is the tallest and how tall is she/he.

b) Who is the heaviest and what is his/her weight.

c) How much heavier is Antonio than Alex?

d) Who are taller than Vanessa and who are shorter?

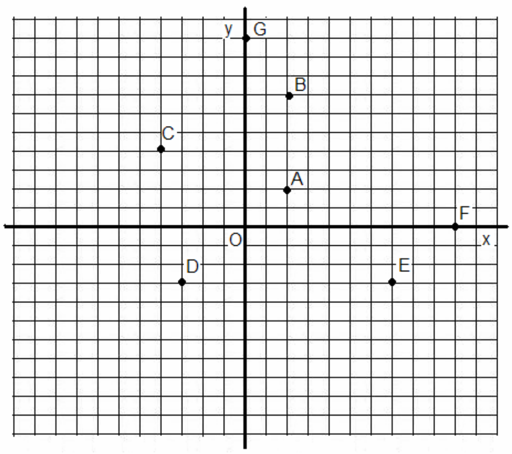
e) In this case, is it true that the taller people are generally heavier?



This is a scatter graph

Exercise 2

a) Write the coordinates of A, B, C, D, E, F, and G



b) Which two points have the same x-coordinate?

c) Which two points have the same y-coordinate?

d) Which points have the same x and y coordinates?

e) Plot a point P which has the same x coordinate as C and the same y coordinate as A.

f) Plot a point, which has the same coordinates as B but in the opposite order.

Exercise 3

a) Plot the points A(-2,-2); B(0,0); C(4,4) and D(5.5,5.5).

b) Draw the line that joins all of them.

c) Plot P(5,7) and P’(7,5). What can you see?

d) Plot Q(-2,4) and Q’(4,-2). What can you see?

The points P’ and Q’ are the images of the points P and Q under reflection of the symmetric line drawn in b).

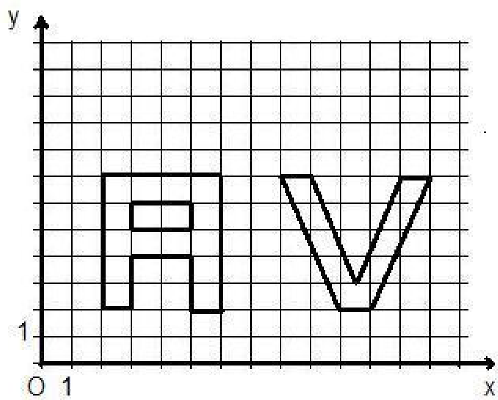
e) Plot the points R(-4,-1) and the symmetric R’

f) Plot the points S(5,2) and the symmetric S’

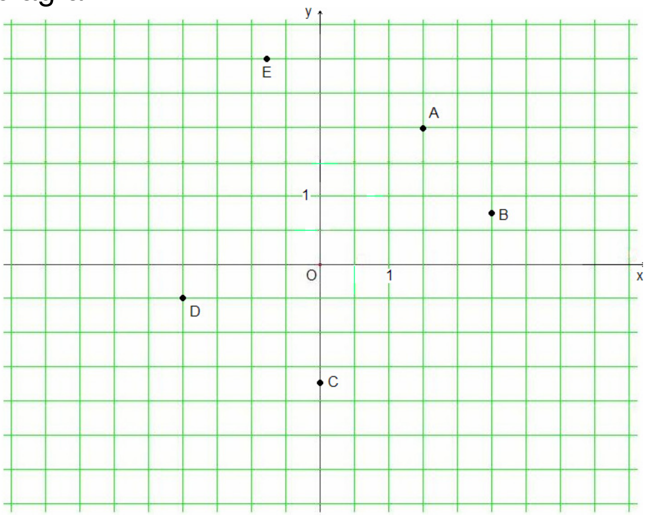
Exercise 4 Draw a grid. Join these points in the order they appear.

1. (2,2); (2,10); (9,10); (9,2); (2,2)
2. (3,7); (3,8); (4,8); (4,7); (3,7)
3. (7,7); (7,8); (8,8); ((8,7); (7,7)
4. (5,5); (5,7); (6,7); (6,5); (5,5)
5. (4,3); (4,4); (7,4); (7,3); (4,3)

Exercise 5 Write a set of instructions to a friend to see if she/he can write the school initials displayed on the grid.



Exercise 6 Write the coordinates of the points that are plotted on the Cartesian diagram.



Exercise 7 For each plotting draw a new coordinate diagram

7.1 Plot the points, join them in order and name the shapes

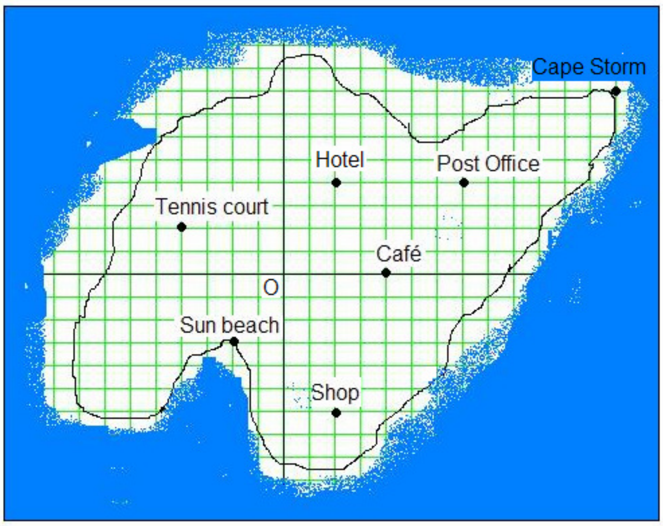
a) (2,2); (2,5); (6,5); (6,2); (2,2)

b) (2,6); (4,4); (7,7); (2,6)

7.2 Write down the coordinates of the missing point and name the shape.
(1,2); (2,5); (5,4); (,)

7.3 Write down the coordinates of the missing points the shape is ABCDEFGH its opposite angles are the same, but the sides are not all of the same length.
A(0,0); B(-2,2); C(-2,4); D(0,6); E(3,6)...

Exercise 8 Look at the map of this small island and:



Scale
1 square = 0.5 km

a) Write the coordinates of every important place in the island.

b) Can Peter walk in a straight line from coordinates (-7,-4) to the shop?

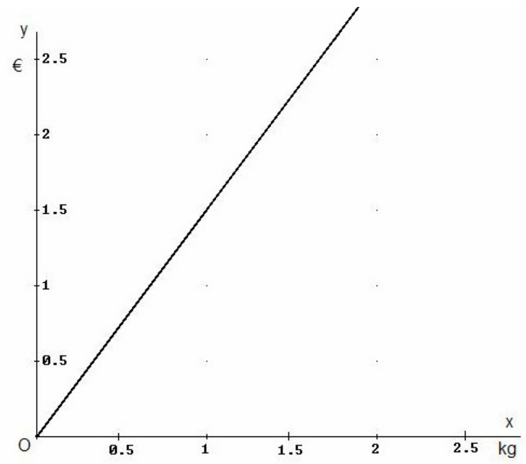
c) Tom is on Sun beach, he travels 3.5 km north and 2 km east. Where is he now?

2 Graph of a function

2.1 Definitions.

Graphs describe relationships between two different quantities; from this relationship we can build up a set of pairs and draw a graph.

Example 4 Tomatoes are sold at €1.5 per kg, we can plot a graph showing the cost depending on the number of kg bought like this:



You can see that all the points are on a line

These magnitudes are directly proportional, when one doubles the second will be doubled also.
In general graphs describe the relationship between two variables x and y

x is the independent variable (in the example kg of tomatoes).

y is the dependent variable (in the example price of the purchase).

We see that y depends on x

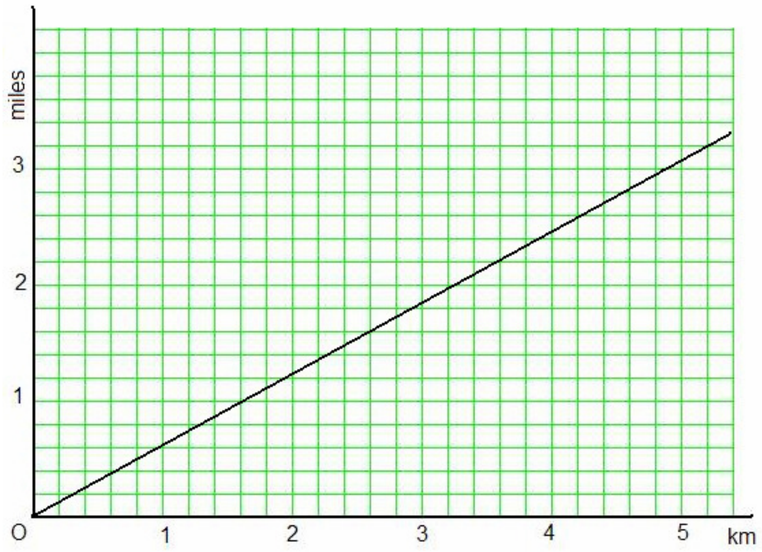
Notice that for each x-value, there is one and only one possible y-value. This is important!

Sometimes it is easy to describe the relationship by a formula in the example the formula is $y = 1.5 \cdot x$

2.1 Conversion graphs.

They are used to convert between different units of a magnitude.

Example 5 converting miles into km and vice versa. We know that 1 mile is approximately 1.609 km and they are directly proportional. We can draw a graph and use it to convert between these units. It is useful using a drawing triangle.

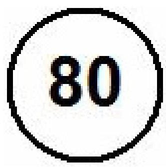


From the graph you can see that 2.5 miles is about 4 km, 3.6 km are about 2.2 miles or 3 km are approximately 1.9 miles.

Can you convert three different lengths in km into miles and three more from miles into km?

Exercises

Exercise 9 If 

in U.K. is the same as 

km/h in Europe

a) Draw a line graph for the conversion between mph and km/h

b) Use the graph to complete the tables:

Speed limit mph In U.K.	Speed limit in km/h In Europe
30 mph	
40 mph	
70 mph	

Speed limit in km/h In Europe	Speed limit mph In U.K.
40	
90	
120	

Exercise 10 Draw a graph to convert € into $ and vice versa. (Use the conversion rate of €10 = 14 $).

Use the graph to convert approximately to the other unit of money:
a) $12 b) €12 c) $20 d) €7 e) $16

Exercise 11 There are two main temperature scales, Fahrenheit which is very common in U.S. and Celsius. These are the data related to the freezing and boiling temperatures of the water.

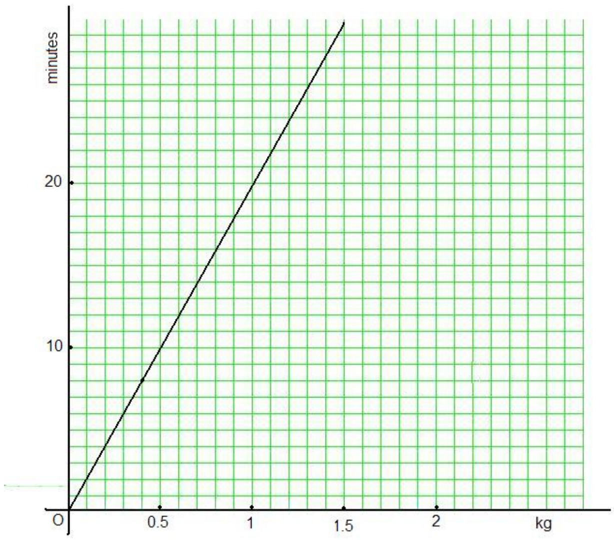
	Freezing point	Boiling point
Celsius	0ºC	100ºC
Fahrenheit	32ºF	212ºF

a) Draw a graph line through the two points given.

b) Use it to convert these temperatures into the other scale.

150ºF	70ºF	90ºF	100ºF	40ºF
15ºC	25ºC	37ºC	40ºC	60C

Exercise 12 The graph below describes the time required to defrost a piece of meat in a microwave oven depending of the weight.



How long is needed to defrost pieces of meat of:

a) 200g

b) 600g

c) 0.5 kg

d) 300g

e) 1kg and 200g

What mass of meat can be defrosted in:

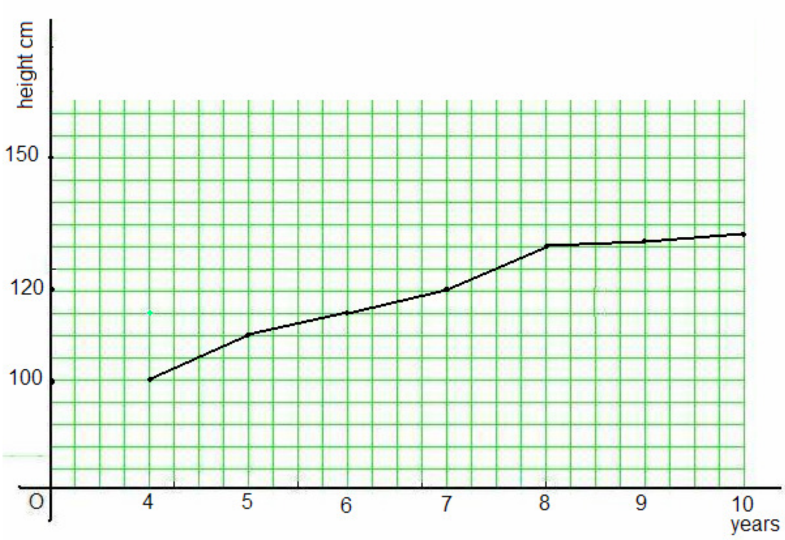
a) 8min

b) 15min

c) 1/2h

How much longer it takes to defrost 300g than 350g of meat?

Exercise 13 This is the height of Wendy recorded every year since she was 4

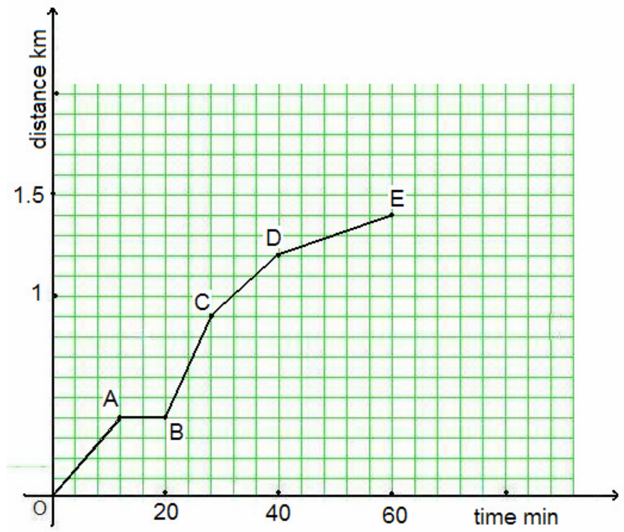


a) How tall was Wendy at 7?

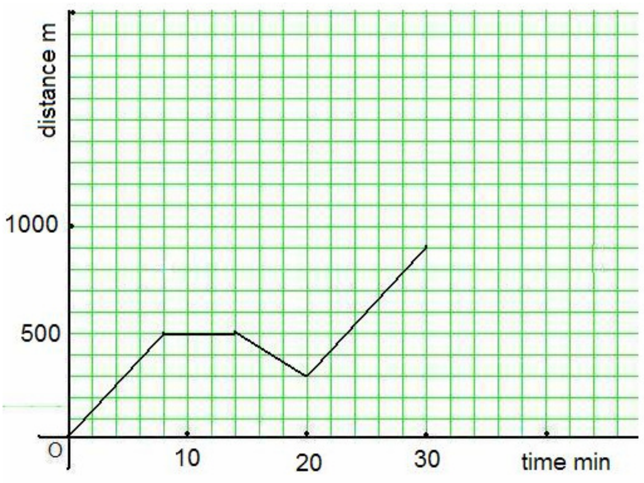
b) How old was she when she reached 132 cm?

c) Between which years she grow faster?

Exercise 14 In this graph we plot the distance travelled by Jane on her way to the school.
Describe the journey.



Exercise 15 The distance from Paul's house to the school is on the graph. Describe his way to school.



3. Handling with data.

When we need to analyse data they must be collected and organized in a table.

Data can be categorical or numerical.

Examples of categorical data are: colour, kind of music, foods, our favourite subject…

Numerical data can be discrete or continuous.

Examples of numerical discrete data are: size of shoes, number of brothers and sisters…

Examples of numerical continuous data are: height, measures of length, …

Exercise 16 Classify as categorical, numerical discrete or numerical continuous the following.

Weight, length of words, favourite sport, number of leaves of a plant, a house number, time practising a sport, favourite TV program, calories of different foods, mark in the last test of maths.

Think of some other types of data on your own and classify them.

3.1 Organising data. Frequency tables

It is advisable to follow these steps.

1. Collect data.

2. Organise data and display them in a frequency table.

3. Draw a graph.

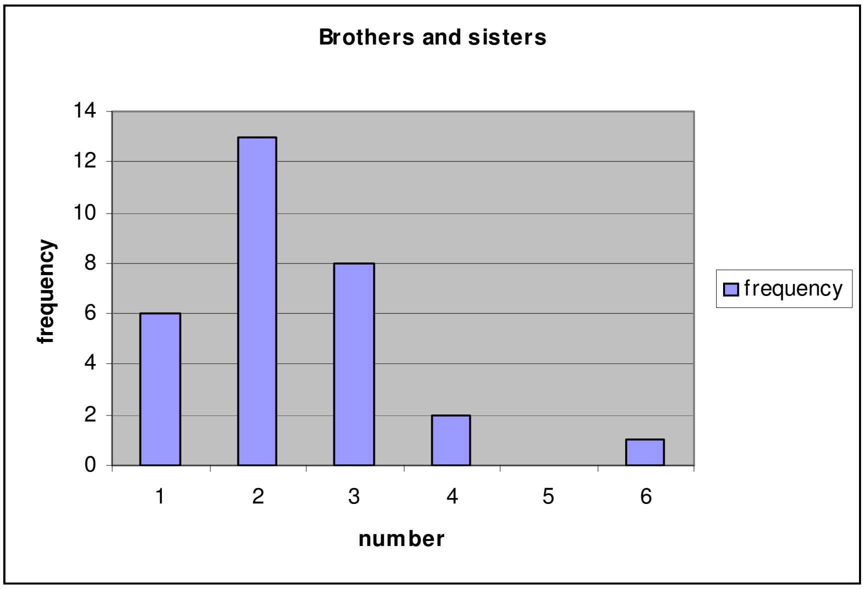
Example 6 30 students have been asked about the number of brothers and sisters they are in their families. These are the answers:

1. Collect data: 1, 2, 1, 3, 6, 3, 2, 1, 1, 1, 2, 2, 3, 2, 3, 2, 2, 4, 2, 3, 3, 2, 2, 3, 4, 2, 2, 3, 1, and 2.

2. Organise data into a frequency table:

Number of B/S	Tally	Frequency
1	IIIII I	6
2	IIIII IIIII III	13
3	IIIII III	8
4	II	2
5		0
6	I	1

Then a bar chart can be drawn with this information.



There are other types of graphs for example a pie chart; in this we draw sectors that represent the proportion in each category.

Example 7 Using the same data from the previous example we can organize them in this way:

Type of family	Frequency
One child	6
Two children	13
Large family	11

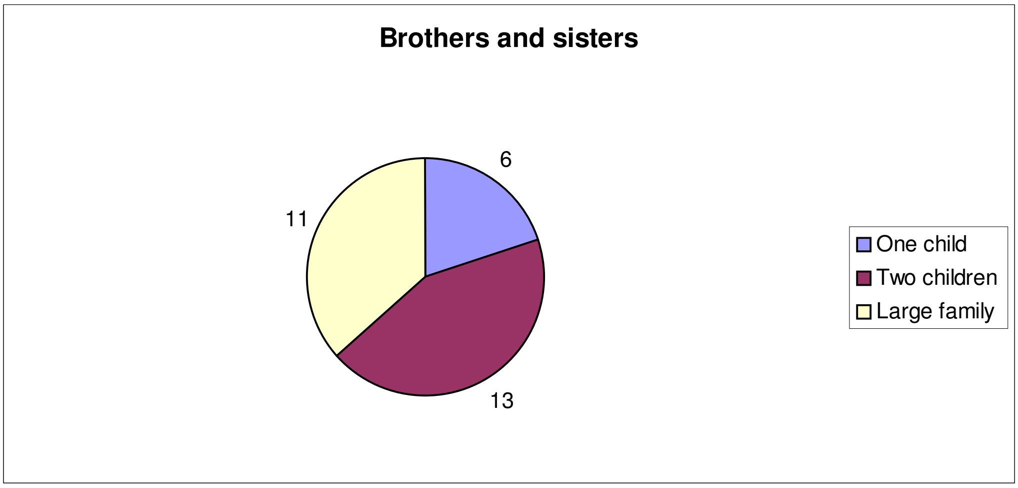
If 30 people is 360º, that is the full circle
6 people would be x and:

$x = \frac{6 \cdot 360}{30} = 72º$, and repeating for the rest of data we get:

Type of family	Frequency	Degrees
One child	6	72º
Two children	13	156º
Large family	11	132º
Total	30	360º

A formula can be used: $\text{Angle} = \frac{\text{frequency} \cdot 360}{\text{Total data}}$

Sectors might be labelled.

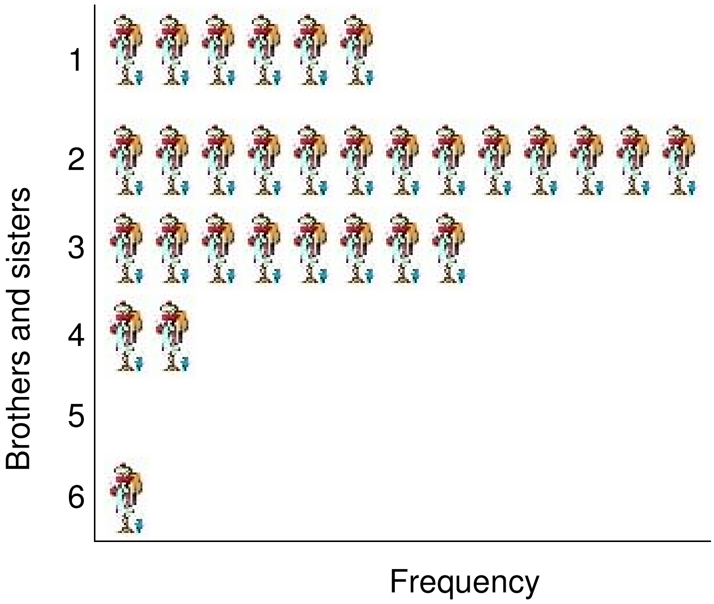


Another type of graph is the pictogram.

Example 8 Using the same data from the example 6 we would have:

Number of brothers and sisters	Frequency
1	6
2	13
3	8
4	2
5	0
6	1

And a pictogram:



Exercises

Exercise 17 Copy and organize in a frequency table the number of vowels in these texts.

- ***"Pure mathematics is in its way, the poetry of logical ideas"*. (Einstein).**
- ***"The laws of Nature are but the mathematical thoughts of God".* (Euclid's).**

Draw a frequency chart and a pie chart with the data.

Translate the two sentences into Spanish and do the same as above.

Exercise 18 Roll a dice 30 times and record the scores

Organize the data in a table

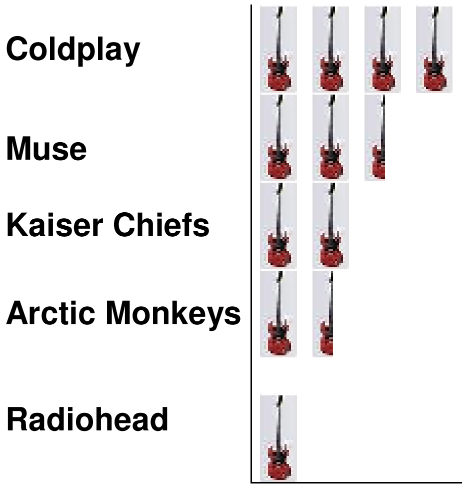
Draw a bar chart

Which is the most frequent score?

Is that what you expected?

How could you improve the reliability of the experiment?

Exercise 19 The results for the best British band survey in a high school are represented in this pictogram



 **Where represents 100 votes**

Organize the data in a table

Draw a pie chart

Exercise 20 The marks in a test of mathematics have been:

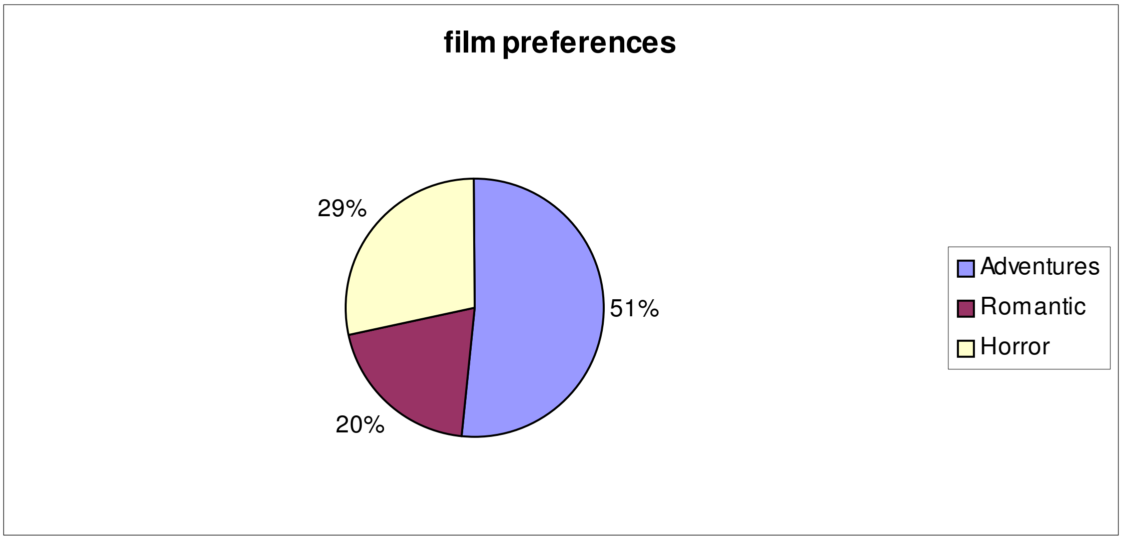
2 3 8 7 7 3 9 5 5 4 1 6 5 7 6 6 5 8 7 3 4 5 6 5 4 6 7 7 9 8

Organize the data in a frequency table

Draw a bar chart

Draw a pie chart organizing the data as IN, SF, B, NT, SB

Exercise 21 In a survey, a group of 80 people were asked which kind of films they liked the most. The answers are represented in this pie chart.



How many people like each kind of film?

Organize the data into a frequency table

Draw a bar chart

Which of the two graphs is the best for describing the data?

Exercise 22 This is the 24-hr average temperature of 118 months between 1981 and 1990 recorded in the Weather station ALBACETE/LOS LLANOS

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year
°C	5.1	6.5	9.5	11.4	15.3	20.9	24.5	23.9	20.8	14.8	9.8	6.3	14.2
°F	41.2	43.7	49.1	52.5	59.5	69.6	76.1	75.0	69.4	58.6	49.6	43.3	57.6

Draw a bar chart with the temperatures in ºC

Draw a line graph with the temperatures in ºF

What can be seen in both graphs?